Application of Multi-Bilinear Binomial Regression Model to Modelling Heterogeneous Mortality Value

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Abstract

A crucial component of mortality modeling is mortality forecasting, which aids in population projections and the calculation of longevity risk for the purpose of actuarial bond pricing. This study examines the Lee-Carter model in the context of diverse populations and suggests a multi-bilinear binomial regression model as an extension of the original model. The models were applied to death rates with a variety of features, and the outcomes were contrasted. The National Bureau of Statistics provided data on male mortality in Nigeria for the age group of five class intervals, spanning 19 years, which the article used for the analysis. Using Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) as measures, the suggested model outperforms the Lee-Carter model in terms of fit to the male mortality rates in Nigeria.

Keywords: Mortality, Lee-Carter, Bilinear, Binomial, AIC

1. Introduction

Mortality refers to the pernicious course by which living members of the population gradationally die out (Oeppen and Vaupel, 2002). The future of human survival and mortality risk has attracted renewed intrigue in modern time. The historic increase in life expectancy shows little sign of slowing and increased survival which is an important contributor to population senescent. The field of mortality and longevity risks and generally, the accurate forecasting and financial control of such risks has become a topic of great interest to academics activities and financial professionals (Edwards and Munhenga, 2011). Over the last centuries significant increases in life expectancy have been observed for the developed countries (Oeppen and Vaupel, 2002). However, this observed decline in mortality rates is not a general pattern, as periods with gender-specific divergence between countries have emerged. In particular in the Scandinavian countries we have observed a decrease in the mortality rates for Norwegian and Swedish females, but a stable or in some cases even increasing mortality rate for Danish females in the period from 1980-1995 (Jacobsen et al. 2004). A similar divergence has not been observed for Scandinavian males. This is an interesting puzzle: why have women in some developed countries had an increasing longevity whereas women in other developed countries have had a stable or decreasing longevity? This phenomena is not unique to the Scandinavian countries. (Meslé and Vallin, 2006) showed that the USA and the Netherlands exhibit a similar slowdown in life expectancy as the Danish women over this period whereas e.g. France and Japan displayed stable positive improvements in life expectancy. The older generation consumes a growing share of their wealth as they age in decades ahead (Marleen, et al, 2014). It is believed that this will put pressure on the company's balance sheets and government pension scheme. It is however observed in (Taruvinga and Gatawa, 2010) that these arrangements of underestimating the population life expectancy, has led to forecasts that are not accurate. The future of mortality is of interest not only for its own usefulness but paramount in the area of population forecasting which is cardinal to economic, social and health planning. The future requirement of health and social security for ageing population is now a considerable assignment for countries to battle with (Alders and De Beer, 2005).

Therefore, the pattern of mortality depends on the distribution of age and sex of each population and mortality rate were thus measured separately for male and female. By mortality rates, this referred to the ratio of deaths to mid-year population size for a given interval of age and time. Mortality rates are among the most important parameters used in evaluating the population health and social levels. In addition to their importance in determining the level of the population's natural growth, Population's growth rate, calculating mortality prospects, creating life tables, Life Insurance and Annuities are products designed to manage financial uncertainty related to how long an individual will survive. Hence, the life time random variable X and its associated mortality models are the basic components in Actuarial Mathematics. In the last 50 years, human mortality has been seen to follow a ceaseless though constantly irregular declined drift, (Lee and Carter 1992). The prospects of longer life are viewed as a positive change for individual and a substantial social achievement but have led to concern over their implications regarding the public spending on old age support and needs.

This research work proposed an extension of Lee-Carter (LC) mortality model to modelling heterogeneous mortality data using Nigeria male mortality data obtained from Nigeria National Bureau of Statistics.

2. Literature Reviewed

To fit and forecast death rates in the US, Lee and Carter introduced a stochastic model in 1992 that was based on a factor analytic method. Numerous individuals have put forth different changes to the LC technique. These include the (Tuljapurkar et al. 2000), (Lee and Miller 2001), and Booth et al. (2002) methods as well as the LC method without adjustment. Booth et al. (2005) assessed the forecast accuracy of the LC approach and its variations for the first time, and Booth et al. (2006) conducted additional research. The LC method has also undergone a number of extensions. Among them, the extension put forth by Hyndman and Ullah, (2007) has drawn more and more interest among Statisticians and Demographers.

To predict mortality and fertility rates, their approach incorporated the concepts of functional principal component regression, nonparametric smoothing, and functional data analysis. Erbas et al. (2007) used this method to predict Australia's breast cancer mortality rates. Hyndman & Booth (2008) have also expanded this approach to incorporate the forecasting of migration rates in Australia and to enhance the variance estimation. It was recently expanded by Hyndman & Shang (2009) to enable the weighting of more recent data. In numerous nations, the Lee and Carter (LC) model has been extensively employed for actuarial and demographic purposes due to its relatively strong performance and ease of use. In numerous nations, the Lee-Carter (LC) model has been extensively employed for actuarial and demographic purposes due to its relatively strong performance and ease of use.

Lee–Carter (LC) model structure is given as in equation 2.1 as,

$$\log m_{x,t} = \alpha_x + b_x k_t + \varepsilon_{xt},\ \textstyle\sum_x b_x = 1,\ \sum_t k_t = 0 \qquad (2.1)$$

where, m_{xt} is the matrix of the central death rates at age X = $(x_1, x_2, - - -, x_n)$ in year t, (t = t_1, t_2- - -,t_{n-1}). The term, ε_{xt}, represents the deviation of the model from the observed log-central death rates and it is expected to be Gaussian Normal i.e $\varepsilon_{xt} \sim N(0, \delta^2)$.

The method assumes a pattern of change in the age distribution of mortality, such that mortality rates decline at different ages maintaining the same time. However, in practice, the general speed of decline at different ages may varies. Wilmoth (1995) opined that the mortality rates at old ages were observed in Sweden to decline more gradually than at other ages 5 to 50 relative to the older and younger ages.

According to Cains et al. (2009), the LC model is unable to produce a non-trivial correlation structure between the year-on-year changes in mortality rates at different ages. The LC model demonstrate mortality development over time for all ages to one single trend, implies perfect correlation between changes which does not seems biologically reasonable.

The LC model does not incorporate cohort effects in the forecasted mortality rates. The incorporation of cohort effect is a desirable property of any stochastic mortality model (Renshaw and Haberman, 2006).

3. Methodology

We propose a modified Lee-Carter model along Renshaw and Haberman model (2006), by including another factor age-Period cohort that will equally preserve the cohort effect and the heterogeneous characteristics of mortality experience in countries with heterogeneous mortality data such as Nigeria by adding an extra bilinear term to Lee-Carter model. This will take care of the effect of age group cohort in the model and the heterogeneous effect not captured in LC model. Hence, the proposed model is given in equation 3.1

$$\mathrm{Log}\left(m_{x,t}\right) = \alpha_x + b_x^{(1)} k_t^{(1)} + b_x^{(2)} k_t^{(2)} + b_x^{(0)} \gamma_{t-x} + E_{xt} \qquad (3.1)$$

Where x is the age variable and t the year under investigation and E_{xt} is the random error.

In order to estimate the model, following Renshaw and Haberman (2006), binomial distribution of deaths was assumed and then used log link function targeting the force of mortality μ_{xt}.

$$\mathrm{L}(\alpha, b_i, k_i) = \ln\left(\prod_t \prod_{x\,w_{xt}} \binom{E_{xt}^0}{\hat{d}_{xt}} \left(1 - \frac{\hat{d}_{xt}}{E_{xt}^0}\right)^{E_{xt}^0 - d_{xt}} \left(\frac{\hat{d}_{xt}}{E_{xt}^0}\right)^{d_{xt}}\right) \qquad (3.2)$$

$$= \sum_t \sum_x w_{xt} \left(\ln\binom{E_{xt}^0}{\hat{d}_{xt}} + (E_{xt}^0 - d_{xt}) ln\left(1 - \frac{\hat{d}_{xt}}{E_{xt}^0}\right) + \left(d_{xt} \ln\left(\frac{\hat{d}_{xt}}{E_{xt}^0}\right)\right)\right) \qquad (3.3)$$

In the model w_{xt} are weights taking the value 0 if a particular (x, t) data cell is omitted or 1 if the cell is present and equation 3.4

$$\hat{d}_{xt} = E_{xt} g^{-1} \sum_t \sum_x \left\{ \left(\alpha_x + b_x^{(1)} k_t^{(1)} + b_x^{(2)} k_t^{(2)} + b_x^0 \gamma_{t-x} \right) \right\} \quad (3.4)$$

Is the expected number of deaths predicted by the model with g^{-1} denoting the inverse of the link function of g. The log likelihood was maximized using Newton Raphson method and because of the presence of the bilinear terms, $b_x^{(1)} k_t^{(1)}, b_x^{(2)} k_t^{(2)}$, the uni-dimensional N Newton method proposed by Goodman (1979) for estimating log-linear models with bilinear terms was implemented. In iteration v+1, a single set of parameters is updated fixing the other parameters at their current estimate using the following updating scheme

$$\hat{\theta}^{(v+1)} = \hat{\theta}^{(v)} - \frac{\frac{dL^{(V)}}{d\theta}}{\frac{d^2 L^{(V)}}{d\theta^2}} \quad (3.5)$$

where, $L^{(V)} = \mathrm{L}(\theta\hat{}((v)\,))$ (3.6)

Using the iterative procedure by setting the starting values for $\hat{a}_x, \hat{b}_x^i, \hat{k}_t^i$ and $\hat{\gamma}_{t-x}$

In our models, there are five *sets* of parameters, these are: $a_x, b_x^{(1)}, k_t^{(1)}, b_x^{(2)}$ and $k_{t-\bar{x}}^{(2)}$ terms. The updating scheme is as follows, starting with $a_x{}^0 = 0$, $b_x^{(0)} = 1, k_t^{(0)} = 0$,

Compute

$$\hat{a}_x^{(v+1)} = \hat{a}_x^{(v)} - \frac{\sum_t \left(D_{xt} - D_{xt}^{(v)} \right)}{\sum_t D_{xt}^{(v)}} \quad (3.7)$$

$$\hat{k}_t^{1(v+1)} = \hat{k}_t^{1(v)} - \frac{\sum_t \left(D_{xt} - D_{xt}^{(v+1)} \right) \hat{b}_x^{1(v)}}{\sum_t D_{xt}^{(v+1)} \left(\hat{b}_x^{1(v)} \right)^2} \quad (3.8)$$

$$\hat{b}_t^{1(v+1)} = \hat{k}_t^{1(v)} - \frac{\sum_t \left(D_{xt} - D_{xt}^{(v+1)} \right) \hat{b}_x^{1(v)}}{\sum_t D_{xt}^{(v+1)} \left(\hat{b}_x^{1(v)} \right)^2} \quad (3.9)$$

$$\hat{b}_x^{2(v+1)} = \hat{b}_x^{2(v)} - \frac{\sum_t \left(D_{xt} - D_{xt}^{(v+1)} \right) \hat{k}_t^{2(v+1)}}{\sum_t D_{xt}^{(v+1)} \left(\hat{k}_t^{2(v+1)} \right)^2} \quad (3.10)$$

$$\hat{k}_t^{2(v+1)} = \hat{k}_t^{2(v)} - \frac{\sum_t \left(D_{xt} - D_{xt}^{(v+1)} \right) \hat{b}_x^{2(v)}}{\sum_t D_{xt}^{(v+1)} \left(\hat{b}_x^{2(v)} \right)^2} \quad (3.11)$$

$$\hat{\gamma}_{t-x}^{1(v+1)} = \hat{\gamma}_{t-x}^{(v)} - \frac{\sum_t \left(D_{xt} - D_{xt}^{(v+1)} \right) \hat{b}_x^{0(v+1)}}{\sum_t D_{xt}^{(v+1)} \left(\hat{b}_{t-x}^{0(v)} \right)^2} \quad (3.12)$$

$$\hat{D}_x^{\left(va_x, vb_x, vk_t, vb_x.vk_t, b_x^0, \gamma_{t-x}^{(v)} \right)} = E_{xt} \quad exp^{\left(\hat{a}_x^{va} + \hat{b}_x^{1(vb_1)} \hat{k}_t^{(1)vk_t} + \hat{b}_x^{2vb_2} \hat{k}_t^{(2)vk_t} + \hat{b}_x^{(0)} \hat{\gamma}_{t-x}^{(v\gamma_{t-x})} \right)} \quad (3.13)$$

Is the estimated number of deaths after va updating the parameter a_x, vb_x, update the parameters, $b_x^{(1)}$, vk_t. Updates the parameter $k_t^{(1)}$, $vb_x^{(2)}$, updates the parameter $b_x^{(2)}$ and vk_t updates the parameter $k_t^{(2)}$ and γ_{t-x}. This is made possible to optimize the binomial likelihood by monitoring the deviance as described in Renshaw and Haberman (2003).

From this updating, we arrived at the mortality model estimate as given in equation 3.14

$$\hat{\eta}_{xt} = \hat{a}_x + \hat{b}_x^{(1)} \hat{k}_t^{(1)} + \hat{b}_x^{(2)} \hat{k}_t^{(2)} + b_x^0 \gamma_{t-x} \quad (3.14)$$

The models were used to analyse the Nigeria male mortality from 1998-2023 as the available mortality data at the time of the research. Hence, the two models to be consider in this research study are Lee-Carter model and Multi bilinear binomial regression model, as presented in equations 3.15 and 3.16 respectively.

$$\text{LC:} \quad \log m_{x,t} = \alpha_x + b_x k_t + \varepsilon_{xt}, \quad (3.15)$$

$$\text{MBBLR;} \quad \mathrm{Log}\left(m_{x,t} \right) = \alpha_x + b_x^{(1)} k_t^{(1)} + b_x^{(2)} k_t^{(2)} + b_x^{(0)} \gamma_{t-x} + E_{xt} \quad (3.16)$$

4. Analysis

In this study, we apply the two models to life mortality data obtained from National Bureau of Statistics covering 25 years of age that is grouped of five years interval. The years under study covered 1998 to 2023. The two models used for the analysis are given in equation 3.15 and 3.16. The two models were applied to Nigeria male mortality data as mentioned above and the discussion of the results, presentations of figures and tables were as presented below.

4.1 Discussion of the Results

Figures 4.1, is the Lee-Carter parameters for male Nigeria mortality rates. From the graphs, it was observed that the α_x slope downward while the coefficient of age $\beta_{x,t}$ slope upward with sig-sag shape. The k_t parameter slope in reverse of the α_x, this is equally visible from tables 4.1 and 4.2 of the parameters. This show that the model produces poor curves and fit for the heterogeneous mortality data. Considering the parameters of the proposed model Multi-bilinear Binomial Regression model (MBBRM) given in figures 4.2, α_x parameter produce a normal curve shape while the cohort parameter portrait a sine shape. k_t^1 and, k_t^2 produce a smoother curves better than the curve produce by LC model for Nigeria male mortality, it was also observed that the curves Produce by the parameter β_x^2 is smoother than LC model given in figure 4.2.

4.3 **Comparison of Accuracy of the Models**

It is commonly expected that models with more parameters will fit the data better when comparing the goodness-of-fit of various models. It is now common in the mortality literature to use information criteria that modify the maximum likelihood criterion by penalizing models with more parameters, thereby ruling out the possibility that the better fit observed in a model is the result of over-parametrization and allowing comparison of the relative performance of several models. The Bayesian Information Criteria (BIC) and the Akaike Information Criteria (AIC) are two of these criteria. The parameters of the two models fitted to the male mortality data from Nigeria are shown in Table 4.3.

We saw that both factors resulted in the same ranking, with the model with the lowest AIC and highest BIC being regarded as the best model. As can be seen from Table 4.3, the multi-bilinear binomial regression model (MBBRM) outperforms the Lee-Carter (LC) model in terms of AIC and BIC. This is particularly evident when contrasting Figure 4.2 of the proposed model with Figure 4.1 of the LC parameters, since the curves generated by the latter are less angular than those by the former. Thus, the suggested model works better than the LC model.

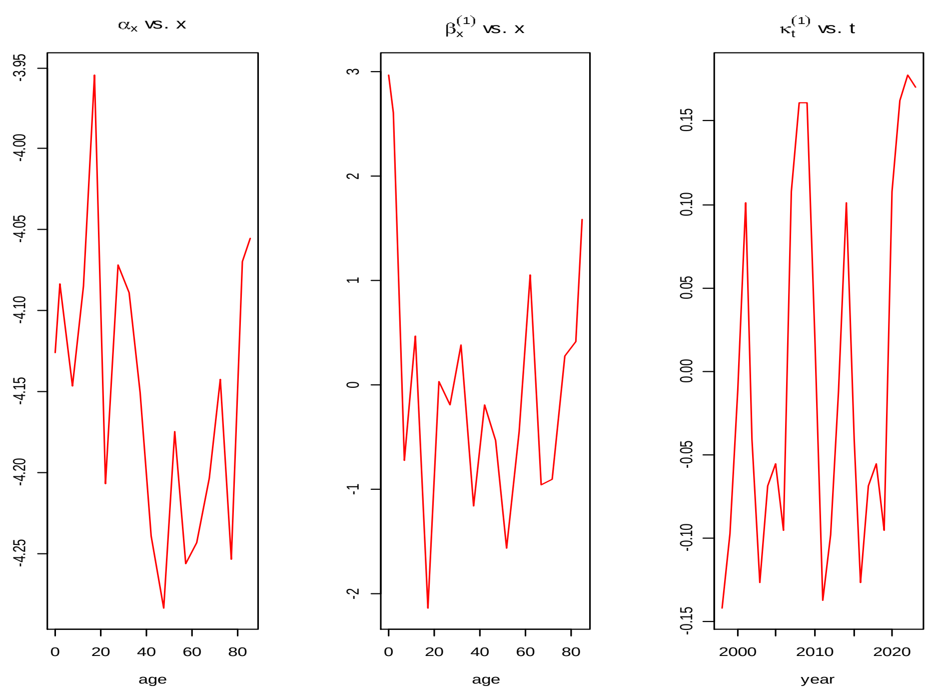

Figure 4.1: LC parameters for Male mortality in Nigeria

The figure 4.1 above, is the plot of the Lee-Carter parameters. The alpha (αx) parameter which isthe scales of the mortality rates across ages producing an undulating shape. Higher alpha parameter means higher mortality rates and lower alpha connotes lower mortality rates. Considering the beta (βx) parameter that modifies the age and represents the rate of change of mortality over time, the curve produces an up and down trends with negative values means an increase in mortality, while positive values means a reduction in mortality value. The kappa (k_t) parameter represents the overall level of mortality. Higher value of k means higher mortality at older age, and lower value of k means lower mortality at lower age.

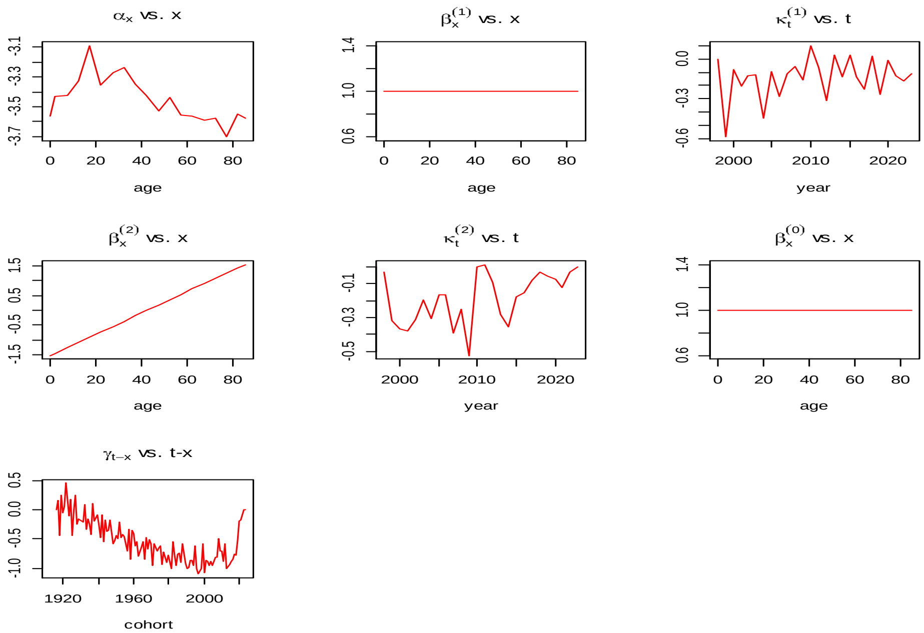

Figure 4.2: MBBRM parameters for Male mortality in Nigeria

Evaluating figure 4.2 above, is the plot of the multi-bilinear parameters. The alpha (αx) parameter which is the scales of the mortality rates across age produces a smother curve than the LC alpha parameters. Higher alpha parameter means higher mortality rates and lower alpha connotes lower mortality rates. Considering the beta (βx) parameter that modifies the age and represents the rate of change of mortality over time, the curve produce straight point curve at point 1 for beta 1 and beta 0, and straight line that transverse from left to right for beta 2. The negative values means an increase in mortality, while positive values means a reduction in mortality value. The kappa (k_t) parameter represents the overall level of mortality. Higher value of k means higher mortality at older age, and lower value of k means lower mortality at lower age.

Table 4.1: Showing the parameters of Lee-Carter and Proposed mortality model

	αx		Bx	
AGE	LC	MBBRM	LC	MBBRM
0	-4.04912	-2.97029	-0.70858	-1.51805
2	-4.09545	-3.83091	-0.49857	-1.44585
7	-4.11037	-6.06391	-1.19791	-1.26536
12	-4.12178	-8.11861	-1.04172	-1.08486
17	-4.159	-9.89117	-0.26485	-0.90437
22	-4.20955	-11.4479	0.338962	-0.72388
27	-3.97416	-12.7028	2.547099	-0.54338
32	-4.1961	-13.6878	-0.1057	-0.36289
37	-4.12784	-14.4124	0.472427	-0.18239
42	-4.01221	-14.8779	-0.14034	-0.0019
47	-4.15991	-15.0977	0.215226	0.178594
52	-4.18654	-15.0584	-0.0042	0.359088
57	-4.25865	-14.7157	0.437976	0.539582
62	-4.15507	-14.1449	0.720789	0.720077
67	-4.24685	-13.3517	0.271844	0.900571
72	-4.16945	-12.2491	-0.36423	1.081065
77	-4.18678	-10.8645	0.206455	1.261559
82	-4.20621	-9.16668	0.450291	1.442053
85	-4.26364	-8.08904	-0.33497	1.55035

Observing the value in table 4.1, it was seen that the alpha parameters of both the LC model and the Multi-bilinear model are negatives with the Multi-bilinear model having the least alpha values showing that there is lower mortality rate at older ages and more convex mortality curve than the LC model. The beta parameters of the two models also show mixed values of positive and negative figures. Looking at the table, it means mortality rates was increase at lower age between ages 0-52 and lower at older ages.

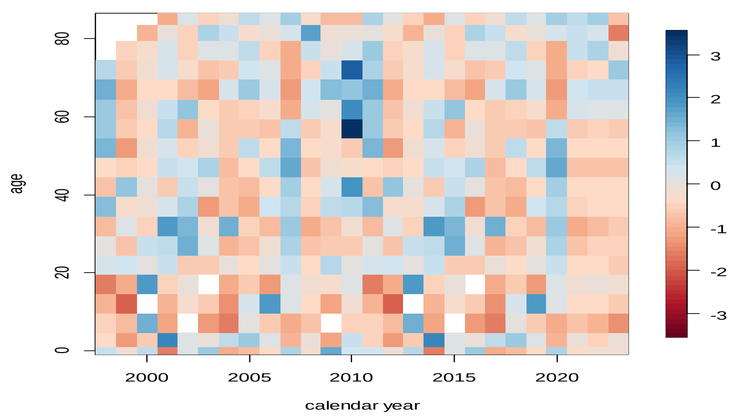

Figure 4.3: Heat maps of deviance residual for Male mortality

Looking at the coloured heat map of the LC model, represents the mortality rates or the life expectancy across different ages and years. From figure 4.3 of the heat map, the warm colours (red, orange), mean higher mortality rates or lower life expectancy, while the blue, green colour, mean lower mortality rate or higher life expectancy. From the curve the red colour dominate the map, meaning there is higher mortality rates as supported by the figure 4.1 and table one above.

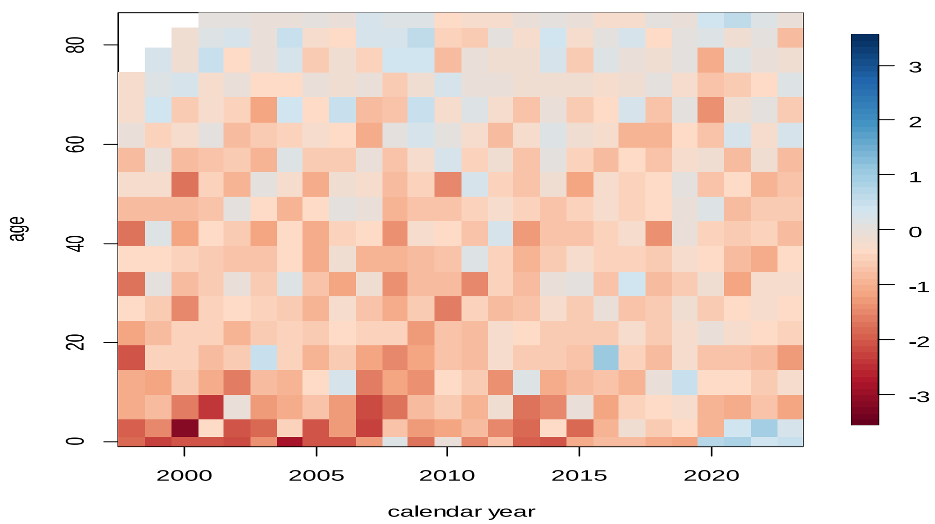

Figure 4.4: Heat maps of deviance residual for MBBRM Male model

Looking at the coloured heat map of the multi-bilinear model, the map represents the mortality rates or the life expectancy across different ages and years. From the curve the red colour dominate the map, meaning there is higher mortality rates as supported by the figure 4.4 and table 4.1 above.

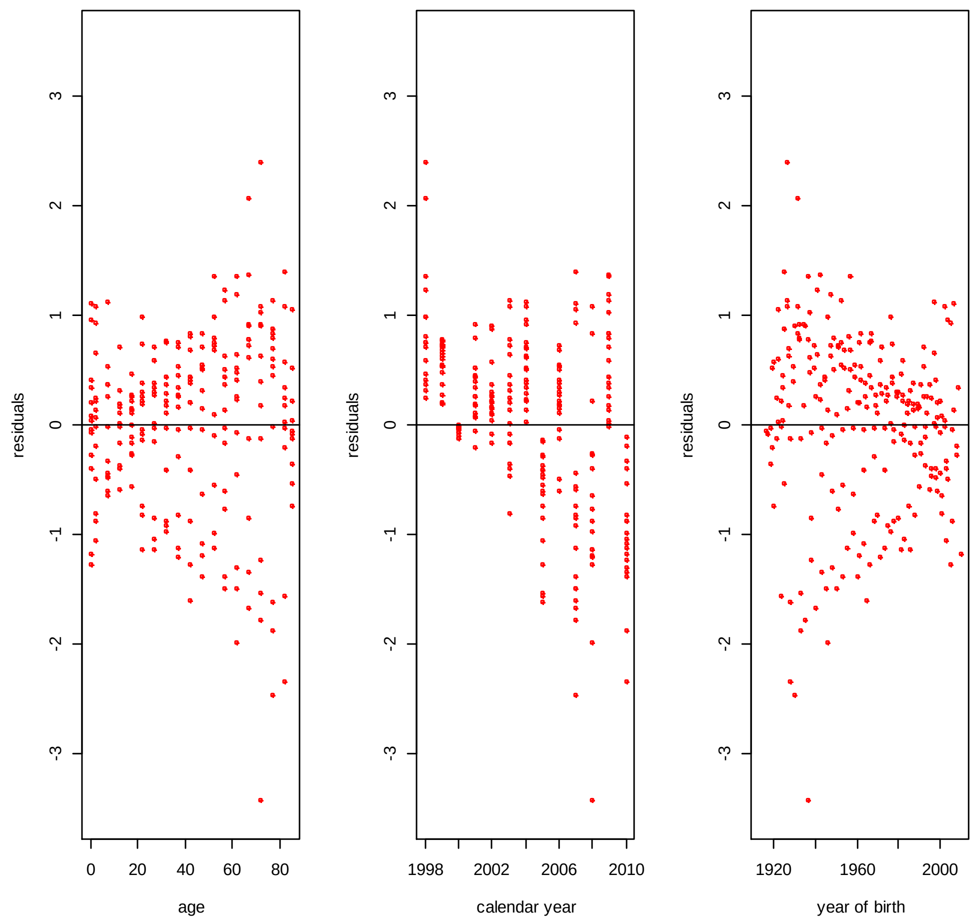

Figure 4.5: Scattered residuals of Lee-Carter Model for Male

The diagrams in figure 4.5 above give the scattered residual of LC model along age, calendar year and year of birth. From the curve, it was observed that residuals scattered across the curve.

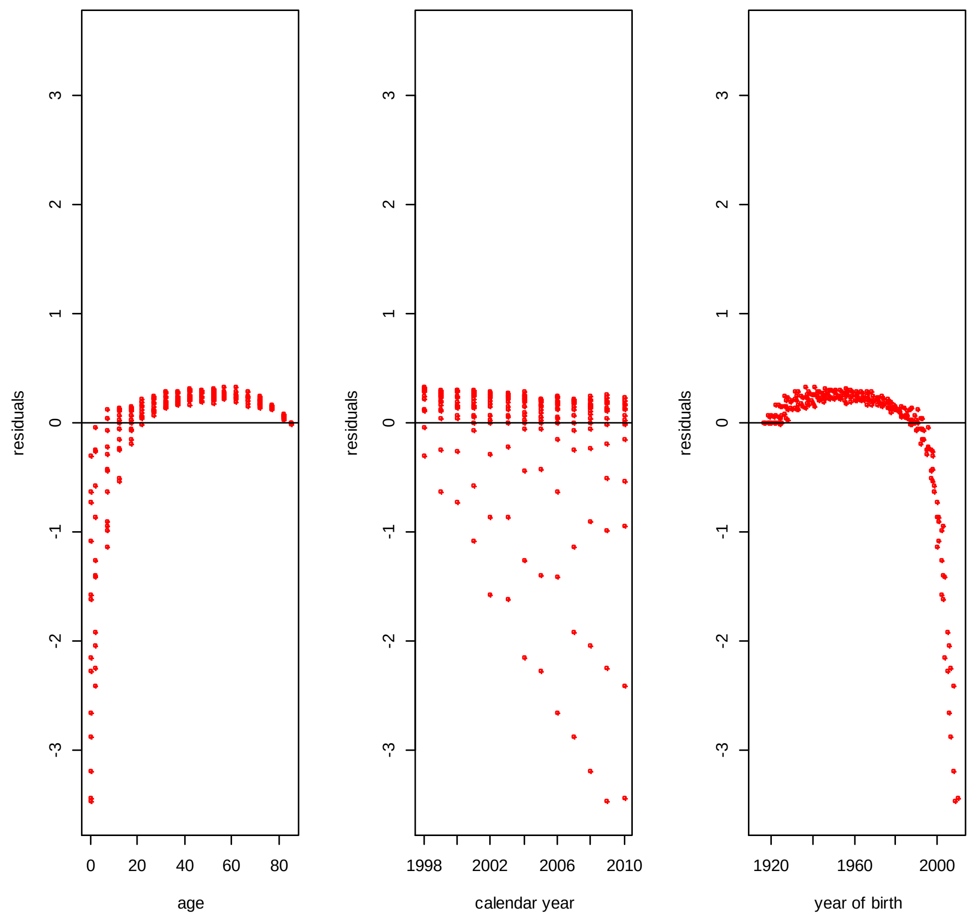

Figure 4.6: Scattered residuals of MBBRM Model for Male

The diagrams in figure 4.6 above give the scattered residual of MBBRM model along age, calendar year and year of birth. From the curve, it was observed that residuals were majorly concentrated on the centre unlike that of LC model that scattered across the curve.

Table 4.2: The k_t parameters of the LC and the MBBRM model

	LC	PM			LC	PM	
YEAR	Kt	$k_t^{(1)}$	$k_t^{(2)}$	YEAR	Kt	$k_t^{(1)}$	$k_t^{(2)}$
1998	-0.4810	1.5865	0.0000	2011	-0.4810	1.5865	0.0000
1999	0.0015	-0.4993	0.0140	2012	0.0015	-0.4993	0.0140
2000	0.1611	-0.1827	-0.0359	2013	0.1611	-0.1827	-0.0359
2001	0.0412	-0.1084	0.0279	2014	0.0412	-0.1084	0.0279
2002	0.1599	-0.4063	-0.0060	2015	0.1599	-0.4063	-0.0060
2003	-0.0067	-0.0600	0.0743	2016	-0.0067	-0.0600	0.0743
2004	0.0456	-0.4467	0.0204	2017	0.0456	-0.4467	0.0204
2005	0.4167	1.3735	0.0180	2018	0.4167	1.3735	0.0180
2006	-0.0930	1.1633	0.0693	2019	-0.0930	1.1633	0.0693
2007	0.0575	0.7472	0.0655	2021	0.0575	0.7472	0.0655
2008	0.0927	0.5315	0.1089	2022	0.0927	0.5315	0.1089
2009	-0.0544	0.1855	0.0431	2023	-0.0544	0.1855	0.0431
2010	-0.3411	0.0000	-0.0154				

Table 4.3: Comparison of Parameters

Models/Parameters		AIC	BIC	DEV	LOG	NO.PARA
LC	MALE	2426.18	2597.24	2156.35	-1114.29	49
MBBRM	MALE	2126.40	2286,24	1049.16	-818.20	175

Table 4.3 is made up of table of comparison for the two models under study using Akaike information criterion (AIC) and Bayesian Information Criterion (BIC) as measures of comparability. It is believe that model with low AIC and BIC is the better model. From the results in table 3, the proposed model which is the multi-bilinear binomial regression model happened to be the model with low AIC (2126.40) and BIC (2286.24). While The Lee-Carter model has AIC of 2426.18 and BIC (2997.24). Hence, the proposed model outperform the LC model based on the values of parameters comparability.

5. Conclusion

The significance and practicality of mortality modeling were highlighted in the study's introduction in Section 1. The study examined the Lee-Carter model and its extensions in Section 2. The analysis of the literature revealed that the majority of the LC model was created in a county with historical data and a constant death rates. As a result, the application of this model to modelling mortality in developing nations with inconsistent mortality data has not been completed.

The Lee-Carter model and the mathematical analysis of the suggested models are briefly described in Section 3. The analysis's findings, the interpretation, discussion, and some consequences were covered in Section 4. The models were applied to modeled male mortality in Nigeria, the suggested model outperformed the LC model, as evidenced by the fact that it provided the lowest AIC and BIC. Furthermore, it is clearer that the suggested model's (MBBRM) curves are smoother than the Lee-Carter model's. It is advised that population statistics be projected for nations with varied death rates, like Nigeria, using the proposed model, based on the study's suggestion and the results obtained.

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