A Modified Friedman Test for Analysis of Randomized Complete Block Design

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Abstract

Friedman test is a non-parametric test that is robust to the departures of normality since it does not depend on the assumption of normality. However, although not readily acknowledged, Friedman test is constrained by the assumption of homogeneity of variance and whenever this assumption is not met, there is generally a loss in the power. In this work, a modified Friedman test that is based on permutation procedure was proposed for the analysis of randomized complete block design (RCBD). The empirical type-I-error rates and powers of the proposed test were compared with that of Friedman test for normal/non-normal errors under equal/unequal variances. The results obtained showed that the modified Friedman test was more robust in terms of type-I-error and more powerful than Friedman test in analysing RCBD for normal/non-normal errors under equal/unequal variances. Hence, the modified Friedman test is highly recommended for the analysis of RCBD.

Keywords: Friedman test, Randomization test, Type-I-error, Power, Analysis of variance, Randomized complete block design.

1. Introduction

Friedman test is a non-parametric test that is robust to the departures of normality since it does not depend on the assumption of normality. However, Friedman test assumes that the observations in blocks are mutually independent and the samples are randomly selected from a given population (O'Gorman, 2001; Laurent & Turk, 2013). Although not readily acknowledged, Friedman test is constrained by the assumption of homogeneity of variance and whenever this assumption is not met, there is generally a loss in the power (Moder, 2010). In order to attempt to solve this problem, modified Friedman test that is based on permutation procedure was proposed for the analysis of randomized complete block design (RCBD).

2. Modified Friedman Test

The modified Friedman test is a distribution-free test that rely on exchangeability of the experimental data in order to construct a reference distribution for the test statistic under the null hypothesis. In hypothesis testing, the modified Friedman test can either be exact or approximate. In the context of ***t***-sample problems, the procedure of the exact form is as follows:

- suppose there are ***N***-observations among t-treatments and n_i observations in treatment $i,\ i = 1, 2, \dots, t$.
- calculate the Friedman statistic for the original data set, called it F_{obs}. All possible permutations are $\frac{N!}{n_i!\, n_2! \dots n_t!}$;
- calculate the Friedman statistic for each permutation to construct a reference distribution for the test statistic.
- obtain the p-value as the proportion of the statistic that are greater than or equal to F_{obs}. This p-value is exact since it was calculated from all possible permutations and it is usually referred to as the significance of the test (Butar, 2008).

As ***N*** increases, the number of permutations can become extremely very large and, consequently, one may decide to take (only need) a random sample of all possible statistic values. In such scenario, the modified Friedman test becomes approximate. In the context of ***t***-sample problems, the procedure of the approximate form is as follows:

- Suppose there are ***N***-observations among t-treatments and n_i observations in treatment $i,\ i = 1, 2, \dots, t$.
- Calculate the Friedman statistic for the original data set, called it F_{obs}.
- Randomly permute the experimental data.
- Compute the Friedman statistic of the permuted data.
- If this Friedman statistic $\geq F_{obs}$, increment the counter.
- Repeat the preceding three step ***n***-times, where ***n*** is the number of iterations.
- Divide the number in the counter by ***n*** to obtain the p-value. This p-value is not exact since it was calculated from a random sample of all possible statistic values (Oladugba et al., 2022).

3. Materials and Methods

3.1 Experimental Data

The data used in this study were simulated from normal and skewed distributions (lognormal, Cauchy and logistic distributions). The simulated data used represents the observed responses of a RCBD experiment adopted from Montgomery (2017, p. 173) which was aimed at examining the effect of three different lubricating oils and five different truck engines on fuel economy. Fuel economy was measured using brake-specific fuel consumption after the engine had run for 15 minutes.

3.2 Friedman Test

Friedman test is a widely used rank-based alternative to the ANOVA F-test for identifying treatment differences in RCBD. The Friedman test does not require that the distribution of the observations across the ***t***-groups within a block are normally distributed. In fact, no particular parametric form is assumed for the distribution of the observations in each group. However, Friedman test assumes that the observations in blocks are mutually independent and the samples are randomly selected from a given population (O'Gorman, 2001; Laurent & Turk, 2013). For a RCBD with ***b***-blocks, ***t***-treatments in each block and one observation per cell; let R_{ij} denote the rank of the response on the ith treatment within block j, where $i = 1, 2, \dots, t\,; j = 1, 2, \dots, b$ and let $R_{i.}$ be sum of the ranks in the ***ith*** treatment. Then, the Friedman test statistic is

$$\boldsymbol{F_{b,t}} = \frac{12\boldsymbol{r}}{\boldsymbol{t}(\boldsymbol{t}+1)} \sum_{\boldsymbol{i}=1}^{\boldsymbol{t}} \left(\overline{\boldsymbol{R}}_{\boldsymbol{i}.} - \overline{\boldsymbol{R}}_{..} \right)^2 \tag{1}$$

where $\bar{R}_{i.}$ and $\bar{R}_{..}$ are the mean rank of the ***ith*** treatment and the overall mean rank of the ***t***-treatments respectively. If ties exist in any block, then the test statistic becomes

$$\boldsymbol{F}^{*}_{\boldsymbol{b.t}} = \frac{\boldsymbol{F}^{*}_{\boldsymbol{b,t}}}{1 - \frac{\sum_{\boldsymbol{r}=1}^{\boldsymbol{k}} \left(\boldsymbol{t}_{\boldsymbol{r}}^{3} - \boldsymbol{t}_{\boldsymbol{r}} \right)}{\boldsymbol{N}\left(\boldsymbol{t}^{2} - 1 \right)}} \tag{2}$$

where, ***k*** is the number of groups with tied observations; t_r is the number of tied observations in the ***ith*** tied group and ***N*** is the total number of observations. If the number of blocks increases infinitely, the distribution of the $F_{b,t}$ approaches χ^2_{t-1} under the null hypothesis in RCBD (Laurent & Turk, 2013). Hence, the null hypothesis is rejected if $F_{b,t} > \chi^2_{\alpha,t-1}$.

3.3 Monte Carlo Simulation

Using Monte Carlo simulation technique, the performance of the proposed test and Friedman test were investigated in RCBD with 4 effect sizes (Δ = 0.5, 0.75, 1 and 1.5), and 5 treatments conditions (***t*** = 3, 6, 9, 12 and 15) that were simultaneously varied with the number of blocks (***b*** = 5, 10, 15, 20, 30, 40 and 50) under 3 variance ratios (σ^2 = 1, 1.5 and 3) when the error is normal or skewed. The standard normal distribution was used for the scenarios under which normality assumption holds while lognormal, Cauchy and logistic distributions were used in cases where the distribution of error was skewed. The R statistical package was used to implement the Monte Carlo technique sampling of 10,000 permutations from the possible $\frac{(tr)!}{(r!)^t}$ permutations for the proposed test (R-test). In the simulation, the experiment was repeated 10000 times for each distribution. In each repetition, the simulated data were analyzed appropriately using the R-test and the Friedman test (FR-test) to obtain the type-I-error rate and power in RCBD. The comparison procedures were considered in two categories. Firstly, if the null hypothesis ($H_0: \tau_i = 0$) was true, the rejection rate of the null hypothesis was considered as the type-I-error rate for each test. The test would been taken to have control over type-I-error rate if type-I-error rates did not exceed the nominal α_0, where α_0 = 0.05. Secondly, if the alternative hypothesis ($H_1: \tau_i \neq 0$) was true, the rejection rate of the null hypothesis was considered as the power for each test. The test that has the larger power would be taken to be more powerful than the other (Oladugba et al., 2022).

Table 1: The 4 scenarios for both normal and skewed distributions

S/N	Type of error	σ^2
1	Normal	Equal
2	Normal	Unequal
3	Skewed	Equal
4	Skewed	Unequal

4. Results

The type-I-error rates and power of FR-Test and R-Test using Monte Carlo simulation were evaluate at a nominal alpha = 5% for t = 3, 6, 9, 12 and 15 under normal/skewed errors with equal/unequal variances. The results obtained for normal error with equal/unequal variances are presented in Tables 2-5 while skewed errors (lognormal, Cauchy and Logistic) with equal/unequal variances are presented in Tables 6-17. In Tables 2, 6, 10 and 14 the values in bold denote the p-values that did not exceed 0.05 while in Tables 3-5, 7-9, 11-13 and 15-17 the values in bold denote the p-values of the test with the higher power.

Normal

It was observed from Table 2 that the type-I-error rates of FR-Test and R-Test had no specific pattern when the number of treatments were varied. Also, for equal/unequal variances, it was observed that the type-I-error rates of FR-Test and R-Test did not exceed 0.05 at t = 6, 9 and 15. Furthermore, FR-Test and R-Test had good control of type-I-error rates when the number of treatments were varied regardless of the variance ratios. The results in Tables 3-5 showed that the power of FR-Test and R-Test increases as number of treatments and effect size increases. For equal variance, the power of FR-Test and R-Test increases as the effect size increases under each treatment but had no specific pattern under unequal variance as the effect size increases. Under equal/unequal variances, the power of R-Test was higher than FR-Test. Moreover, under unequal variance, the power of FR-Test was higher than R-Test at t = 12 and 15.

Lognormal

The results presented in Table 6 showed that the type-I-error rates of FR-Test and R-Test had no specific pattern when the number of treatments were varied. Furthermore, for equal/unequal variances, it was observed that the type-I-error rates of FR-Test and R-Test did not exceed 0.05 at t = 6, 9 and 12. Also, the type-I-error rates of FR-Test was more conservative than that of R-test under unequal variance. Moreover, the type-I-error rates of FR-Test was approximately the same with that of R-Test under equal variance. The results in Tables 7-9 showed that the power of FR-Test and R-Test had no specific pattern when the effect sizes were varied under each treatment. Under equal/unequal variances, the power of R-Test was higher than FR-Test. Moreover, under unequal variance, the power of FR-Test was higher than R-Test at t = 15.

Cauchy

The results in Table 10 showed that the type-I-error rates of FR-Test and R-Test had no specific pattern when the number of treatments were varied. Also, for equal/unequal variances, it was observed that FR-Test and R-Test were having type-I-error rates below the nominal alpha of 5% under each treatment. Moreover, the type-I-error rates of FR-Test and R-Test exceeded 0.05 at t = 3 and 12. The results presented in Tables 7-9 showed that the power of FR-Test and R-Test had no specific pattern when the effect sizes were varied under each treatment. Under equal/unequal variances, the power of R-Test was higher than FR-Test. Moreover, under unequal variance, the power of FR-Test was higher than R-Test at t = 15.

Logistic

The results in presented in Table 14 showed that the type-I-error rates of FR-Test and R-Test had no specific pattern when the number of treatments were varied. Also, for equal/unequal variances, it was observed that the type-I-error rates of FR-Test and R-Test were approximately the same. The results in Tables 15-17 showed that the showed that the power of FR-Test and R-Test increases as number of treatments and effect size increases. Regardless of the variance ratios, the power of R-Test was higher than FR-Test under each treatment.

5. Discussion of Results

For normal error, it was observed that the type-I-error rates of FR-Test and R-Test had good control of type-I-error rates when the number of treatments were varied regardless of the variance ratios. This implies that the two tests were robust to the violation of constant variance assumption. Furthermore, for equal/unequal variances, the type-I-error rates of FR-Test and R-Test did not exceed 0.05 at t = 6, 9 and 15. For data sampled from lognormal, Cauchy and logistic distributions, FR-Test and R-Test were observed to had good control of type-I-error rates when the number of treatments were varied. This implies that FR-Test and R-Test were robust to the violation of normality assumption. For skewed error (lognormal and logistic) under equal/unequal variances, R-Test gave more reliable results than FR-Test. Although, under skewed error (Cauchy), the type-I-error rates of FR-Test was more conservative than that of R-test under unequal variance. It is important to note that based on the simulation results, the type-I-error rates of FR-Test and R-Test were approximately the same regardless of the variance ratios and the distribution of the error.

For normal error under equal variance, the power of FR-Test and R-Test were observed to increase as the number of treatments and effect size increases. Moreover, under unequal variance, the power of FR-Test and R-

Test had no specific pattern as the effect size increases under each treatment. Furthermore, the R-Test was observed to be more powerful than FR-Test as the number of treatments and effect size were varied under equal/unequal variances. Generally, the power of FR-Test and R-Test decreases as the variance ratio increases for each treatment and effect size under normal error. For skewed error (lognormal and Cauchy), the power of FR-Test and R-Test had no specific pattern as the number of treatments and effect size increases. Moreover, for data sampled from logistic distribution, the power of FR-Test and R-Test increases as number of treatments and effect size increases. Also, for skewed error (lognormal, Cauchy and logistic) under equal variance, R-Test was observed to be more powerful than FR-Test. For unequal variance, the R-Test was observed to be more powerful than FR-Test when the data were sampled from Cauchy and logistic distributions. Moreover, under unequal variance, the power of FR-Test was higher than R-Test at $\boldsymbol{t} = 15$ under lognormal distribution.

6. Conclusion

This work proposed a modified Friedman test for the analysis of RCBD and compared the empirical type-I-error rates and powers of the proposed test with that of Friedman test for normal/skewed errors under equal/unequal variances in RCBD using Monte Carlo simulation. The results obtained showed that the modified Friedman test was more robust in terms of type-I-error and more powerful than Friedman test in analysing RCBD for normal/skewed errors under equal/unequal variances. Hence, the modified Friedman test is highly recommended for the analysis of RCBD.

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Table 2: Type-I-error rate of Friedman test and randomization test under normal error at α = 0.05

Number of Treatments	Number of blocks	$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
		FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
$t = 3$	5	**0.0392**	**0.0407**	**0.0289**	**0.0397**	**0.0226**	**0.0399**
	10	**0.0445**	**0.0458**	**0.0483**	**0.0460**	**0.0489**	**0.0464**
	15	0.0525	**0.0440**	0.0514	**0.0448**	**0.0489**	**0.0454**
	20	0.0506	0.0564	**0.0499**	0.0550	**0.0487**	0.0550
	30	**0.0496**	0.0564	**0.0482**	0.0550	**0.0480**	0.0575
	40	0.0517	**0.0499**	0.0528	0.0505	0.0535	**0.0489**
	50	**0.0487**	**0.0480**	**0.0489**	**0.0450**	**0.0493**	**0.0465**
$t = 6$	5	**0.0415**	**0.0271**	**0.0411**	**0.0293**	**0.0416**	**0.0296**
	10	**0.0444**	**0.0372**	**0.0483**	**0.0362**	**0.0467**	**0.0364**
	15	**0.0409**	**0.0397**	**0.0410**	**0.0403**	**0.0412**	**0.0407**
	20	**0.0401**	**0.0373**	**0.0396**	**0.0366**	**0.0395**	**0.0380**
	30	**0.0411**	**0.0393**	**0.0424**	**0.0406**	**0.0417**	**0.0396**
	40	**0.0447**	**0.0382**	**0.0461**	**0.0387**	**0.0450**	**0.0400**
	50	**0.0401**	**0.0409**	**0.0415**	**0.0415**	**0.0419**	**0.0410**
$t = 9$	5	**0.0477**	**0.0257**	**0.0485**	**0.0245**	**0.0470**	**0.0264**
	10	**0.0446**	**0.0395**	**0.0454**	**0.0402**	**0.0447**	**0.0382**
	15	**0.0418**	**0.0421**	**0.0428**	**0.0413**	**0.0429**	**0.0416**
	20	**0.0414**	**0.0416**	**0.0420**	**0.0408**	**0.0431**	**0.0407**
	30	**0.0455**	**0.0427**	**0.0459**	**0.0447**	**0.0471**	**0.0444**
	40	**0.0470**	**0.0402**	**0.0460**	**0.0407**	**0.0496**	**0.0424**
	50	**0.0426**	**0.0428**	**0.0447**	**0.0435**	**0.0432**	**0.0437**
$t = 12$	5	0.0514	**0.0298**	**0.0494**	**0.0307**	**0.0470**	**0.0293**
	10	**0.0467**	**0.0389**	**0.0457**	**0.0392**	**0.0468**	**0.0381**
	15	**0.0465**	**0.0405**	**0.0448**	**0.0410**	**0.0463**	**0.0396**
	20	**0.0456**	**0.0435**	**0.0457**	**0.0446**	**0.0451**	**0.0428**
	30	**0.0446**	**0.0446**	**0.0462**	**0.0447**	**0.0455**	**0.0446**
	40	**0.0482**	**0.0443**	**0.0460**	**0.0436**	**0.0458**	**0.0430**
	50	**0.0485**	**0.0467**	**0.0490**	**0.0473**	**0.0491**	**0.0474**
t = 15	5	**0.0469**	**0.0332**	**0.0418**	**0.0322**	**0.0436**	**0.0324**
	10	**0.0488**	**0.0424**	**0.0492**	**0.0410**	**0.0489**	**0.0402**
	15	**0.0462**	**0.0449**	**0.0470**	**0.0456**	**0.0476**	**0.0445**
	20	**0.0442**	**0.0407**	**0.0459**	**0.0403**	**0.0420**	**0.0421**
	30	**0.0439**	**0.0453**	**0.0453**	**0.0451**	**0.0453**	**0.0443**
	40	**0.0444**	**0.0453**	**0.0455**	**0.0444**	**0.0462**	**0.0454**
	50	**0.0449**	**0.0443**	**0.0438**	**0.0437**	**0.0441**	**0.0432**

Table 3: Power of the Friedman test and randomization test under normal error for t = 3 and 6

α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	3(5)	0.3024	0.6305	0.3784	**0.8154**	0.1877	**0.4789**
	3(10)	0.7505	**0.9890**	0.7505	**0.9881**	0.2831	**0.6870**
	3(15)	0.8868	**0.9980**	0.8592	**0.9973**	0.6233	**0.9599**
Δ = 0.50	3(20)	0.5204	**0.9238**	0.9409	**0.9451**	0.2160	**0.5592**
	3(30)	0.8099	**0.9945**	0.8737	**0.9980**	0.2319	**0.6279**
	3(40)	0.6780	**0.9725**	0.4909	**0.9124**	0.3970	**0.8519**
	3(50)	0.7355	**0.9838**	0.6223	**0.9617**	0.2217	**0.5968**
	3(5)	0.3024	0.6272	0.3784	**0.8179**	0.1877	**0.4789**
	3(10)	0.8656	**0.9980**	0.8656	**0.9984**	0.5037	**0.9091**
	3(15)	0.8868	**0.9981**	0.8868	**0.9978**	0.6510	**0.9720**
Δ = 0.75	3(20)	0.7755	**0.9891**	0.5916	**0.9475**	0.2351	**0.6188**
	3(30)	0.9616	**0.9998**	0.8810	**0.9976**	0.3970	**0.8391**
	3(40)	0.9589	**0.9996**	0.8656	**0.9967**	0.4909	**0.9114**
	3(50)	0.8907	**0.9981**	0.8002	**0.9916**	0.4900	**0.9065**
	3(5)	0.3784	**0.8110**	0.3024	0.6311	0.1507	**0.3098**
	3(10)	0.8656	**0.9977**	0.8656	**0.9982**	0.5991	**0.9562**
	3(15)	0.8937	**0.9986**	0.8868	**0.9784**	0.6938	**0.9777**
Δ = 1.00	3(20)	0.9599	**0.9999**	0.6938	**0.9784**	0.2351	**0.6227**
	3(30)	0.9870	**1**	0.9689	**0.9999**	0.5529	**0.9378**
	3(40)	0.9944	**1**	0.9215	**0.9987**	0.5991	**0.9527**
	3(50)	0.9888	**1**	0.9071	**0.9982**	0.6763	**0.9757**
	3(5)	0.5201	**0.9027**	0.3784	**0.8098**	0.1507	**0.3084**
	3(10)	0.9165	**0.9995**	0.8656	**0.9973**	0.8312	**0.9960**
	3(15)	0.9623	**0.9999**	0.9065	**0.9990**	0.7399	**0.9845**
Δ = 1.50	3(20)	0.9891	**1**	0.9764	**0.9999**	0.3311	**0.7694**
	3(30)	0.9998	**1**	0.9983	**1**	0.8312	**0.9949**
	3(40)	**1**	**1**	0.9985	**1**	0.8134	**0.9930**
	3(50)	**1**	**1**	0.9753	**1**	0.8579	**0.9970**
α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	6(5)	0.6364	**0.9149**	0.6364	**0.9142**	0.5904	**0.8832**
	6(10)	0.6284	**0.9056**	0.6316	**0.9061**	0.2243	**0.3103**
	6(15)	0.8625	**0.9943**	0.8625	**0.9935**	0.7427	**0.9710**
Δ = 0.50	6(20)	0.7720	**0.9724**	0.7386	**0.9616**	0.3262	**0.5410**
	6(30)	0.6824	**0.9400**	0.6525	**0.9276**	0.2165	**0.3119**
	6(40)	0.4854	**0.7843**	0.3899	0.6588	0.1825	**0.2244**
	6(50)	0.6739	**0.9304**	0.5445	**0.8350**	0.2134	**0.2968**
	6(5)	0.6904	**0.9482**	0.6671	**0.9340**	0.6427	**0.9224**
	6(10)	0.7112	**0.9477**	0.6535	**0.9192**	0.2877	**0.4494**
	6(15)	0.8941	**0.9962**	0.8645	**0.9940**	0.7223	**0.9629**
Δ = 0.75	6(20)	0.8642	**0.9910**	0.7709	**0.9710**	0.4564	**0.7464**
	6(30)	0.7784	**0.9736**	0.7577	**0.9686**	0.2191	**0.3203**
	6(40)	0.7258	**0.9531**	0.4921	0.7944	**0.1429**	0.1346
	6(50)	0.8859	**0.9934**	0.7649	**0.9676**	0.3290	**0.5381**
	6(5)	0.6904	**0.9489**	0.6904	**0.9488**	0.6671	**0.9385**
	6(10)	0.8243	**0.9899**	0.7030	**0.9430**	0.2877	**0.4489**
	6(15)	0.9682	**0.9997**	0.8941	**0.9955**	0.7258	**0.9655**
Δ = 1.00	6(20)	0.9480	**0.9988**	0.8845	**0.9930**	0.5430	**0.8412**
	6(30)	0.9677	**0.9997**	0.8149	**0.9799**	0.2837	**0.4661**
	6(40)	0.9505	**0.9987**	0.7051	**0.9422**	**0.1508**	0.1482
	6(50)	0.9761	**0.9996**	0.9223	**0.9971**	0.4926	**0.7757**
	6(5)	0.7232	**0.9629**	0.7232	**0.9636**	0.6904	**0.9504**
	6(10)	0.9336	**0.9988**	0.8715	**0.9945**	0.4329	**0.7001**
	6(15)	0.9947	**1**	0.9713	**0.9996**	0.7292	**0.9652**
Δ = 1.50	6(20)	0.9974	**1**	0.9782	**0.9996**	0.7411	**0.9597**
	6(30)	0.9992	**1**	0.9870	**1**	0.4421	**0.7281**
	6(40)	0.9999	**1**	0.9877	**1**	0.4200	**0.6894**
	6(50)	0.9999	**1**	0.9996	**1**	0.7695	**0.9657**

Table 4: Power of the Friedman test and randomization test under normal error for t = 9 and 12

α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	9(5)	0.5195	**0.6287**	0.4610	**0.5183**	0.4483	**0.4969**
	9(10)	0.8631	**0.9827**	0.8248	**0.9663**	0.7672	**0.9380**
	9(15)	0.8143	**0.9563**	0.8122	**0.9538**	0.4910	**0.5942**
Δ = 0.50	9(20)	0.6009	**0.7594**	0.5119	**0.6385**	**0.2422**	0.1584
	9(30)	0.8894	**0.9857**	0.8543	**0.9752**	**0.3396**	0.3378
	9(40)	0.5531	**0.7032**	0.4809	**0.5881**	**0.2239**	0.1283
	9(50)	0.6572	**0.8321**	0.6744	**0.8485**	**0.2481**	0.1643
	9(5)	0.5579	**0.6917**	0.5104	**0.6076**	0.4101	**0.4258**
	9(10)	0.8758	**0.9856**	0.8397	**0.9712**	0.7643	**0.9363**
	9(15)	0.8936	**0.9865**	0.8621	**0.9764**	0.5275	**0.6495**
Δ = 0.75	9(20)	0.7022	**0.8751**	0.5679	**0.7191**	**0.2179**	0.1172
	9(30)	0.9758	**0.9995**	0.9695	**0.9996**	0.6043	**0.7755**
	9(40)	0.8102	**0.9517**	0.6036	**0.7694**	**0.2538**	0.1766
	9(50)	0.9188	**0.9906**	0.7848	**0.9373**	0.3694	**0.3865**
	9(5)	0.5835	**0.7289**	0.5074	**0.6020**	0.4357	**0.4732**
	9(10)	0.9138	**0.9945**	0.8705	**0.9823**	0.7691	**0.9370**
	9(15)	0.9450	**0.9966**	0.8986	**0.9871**	0.5675	**0.7063**
Δ = 1.00	9(20)	0.8470	**0.9699**	0.6617	**0.8335**	**0.2171**	0.1172
	9(30)	0.9974	**1**	0.9930	**1**	0.7434	**0.9108**
	9(40)	0.9626	**0.9989**	0.8539	**0.9729**	**0.2845**	0.2290
	9(50)	0.9943	**1**	0.9435	**0.9955**	0.4518	**0.5318**
	9(5)	0.6191	**0.7832**	0.5722	0.7116	0.4672	**0.5281**
	9(10)	0.9744	**0.9995**	0.9297	**0.9959**	0.7928	**0.9509**
	9(15)	0.9789	**0.9993**	0.9692	**0.9986**	0.6367	**0.7972**
Δ = 1.50	9(20)	0.9731	**0.9995**	0.9211	**0.9920**	**0.3495**	0.3462
	9(30)	**1**	**1**	0.9997	**1**	0.9338	**0.9942**
	9(40)	0.9998	**1**	0.9949	**1**	0.5586	**0.7075**
	9(50)	**1**	**1**	0.9997	**1**	0.7479	**0.9096**
α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	12(5)	**0.5110**	0.4199	**0.4578**	0.3245	**0.4676**	0.3422
	12(10)	0.8499	**0.9418**	0.8470	**0.9403**	0.7486	**0.8455**
	12(15)	0.7778	**0.8756**	0.7408	**0.8288**	**0.4170**	0.2920
Δ = 0.50	12(20)	**0.4500**	0.3346	**0.5110**	0.4430	**0.3111**	0.1297
	12(30)	0.6150	**0.6380**	0.6273	**0.6594**	**0.2507**	0.0636
	12(40)	0.7963	**0.8846**	0.7916	**0.8800**	**0.4508**	0.3470
	12(50)	0.8284	**0.9239**	0.7110	**0.7895**	**0.3080**	0.1310
	12(5)	**0.5499**	0.4921	**0.5062**	0.4096	**0.4676**	0.3390
	12(10)	0.8735	**0.9609**	0.8621	**0.9517**	0.7607	**0.8596**
	12(15)	0.8288	**0.9246**	0.7823	**0.8803**	**0.4165**	0.2899
Δ = 0.75	12(20)	**0.5800**	0.5658	**0.4459**	0.3270	**0.2885**	0.1015
	12(30)	0.6862	**0.7498**	0.6511	**0.6975**	**0.2256**	0.0423
	12(40)	0.9590	**0.9945**	0.9612	**0.9955**	**0.4676**	0.3390
	12(50)	0.9613	**0.9955**	0.8575	**0.9462**	**0.4058**	0.2803
	12(5)	**0.6147**	0.6133	**0.5575**	0.5066	**0.4854**	0.3726
	12(10)	0.9036	**0.9776**	0.8805	**0.9647**	0.7668	**0.8671**
	12(15)	0.9010	**0.9728**	0.8061	**0.9064**	**0.4714**	0.3830
Δ = 1.00	12(20)	0.7070	**0.7672**	**0.5560**	0.5203	**0.2424**	0.0559
	12(30)	0.8828	**0.9627**	0.7574	**0.8474**	**0.2190**	0.0382
	12(40)	0.9976	**1**	0.9938	**1**	0.8057	**0.8969**
	12(50)	0.9984	**1**	0.9762	**0.9981**	**0.4942**	0.4320
	12(5)	0.7437	**0.8303**	0.6940	**0.7524**	**0.5330**	0.4632
	12(10)	0.9761	**0.9988**	0.9187	**0.9838**	0.7893	**0.8903**
	12(15)	0.9818	**0.9995**	0.9478	**0.9930**	**0.5716**	0.5644
Δ = 1.50	12(20)	0.9582	**0.9957**	0.8444	**0.9288**	**0.2942**	0.1082
	12(30)	0.9964	**0.9999**	0.9494	**0.9939**	**0.2809**	0.0928
	12(40)	**1**	**1**	0.9999	**1**	0.9613	**0.9946**
	12(50)	**1**	**1**	**1**	**1**	0.8342	**0.9281**

Table 5: Power of the Friedman test and randomization test under normal error for $\boldsymbol{t}$ = 15

α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	15(5)	**0.7345**	0.7084	**0.7207**	0.6790	**0.7379**	0.7122
	15(10)	0.8449	**0.8825**	0.8377	**0.8709**	**0.7732**	0.7729
	15(15)	0.7670	**0.7673**	**0.7158**	0.6782	**0.4130**	0.1485
Δ = 0.50	15(20)	**0.6005**	0.4564	**0.6018**	0.4583	**0.3883**	0.1247
	15(30)	**0.6199**	0.5038	**0.5952**	0.4563	**0.3015**	0.0480
	15(40)	**0.6334**	0.5268	**0.5722**	0.4187	**0.3444**	0.0869
	15(50)	0.8593	**0.8944**	0.8540	**0.8874**	**0.4793**	0.2530
	15(5)	**0.7399**	0.7178	**0.7235**	0.6825	**0.7372**	0.7124
	15(10)	0.8384	**0.8726**	0.8241	**0.8536**	**0.7618**	0.7512
	15(15)	0.8593	**0.9035**	0.8492	**0.8902**	**0.4405**	0.1824
Δ = 0.75	15(20)	**0.6620**	0.5719	**0.6454**	0.5401	**0.3816**	0.1148
	15(30)	0.7626	**0.7666**	**0.6852**	0.6231	**0.3205**	0.0617
	15(40)	0.8177	**0.8403**	**0.6869**	0.6266	**0.3619**	0.1024
	15(50)	0.9767	**0.9954**	0.9378	**0.9760**	**0.5902**	0.4407
	15(5)	0.7795	**0.7892**	**0.7438**	0.7224	**0.7637**	**0.7637**
	15(10)	0.8588	**0.9043**	0.8289	**0.8599**	**0.7432**	0.7201
	15(15)	0.9184	**0.9634**	0.9166	**0.9632**	**0.5086**	0.2911
Δ = 1.00	15(20)	**0.7789**	0.7786	0.7422	0.7172	**0.3893**	0.1224
	15(30)	0.8957	**0.9450**	0.8003	**0.8255**	**0.3721**	0.1088
	15(40)	0.9695	**0.9918**	0.8707	**0.9069**	**0.3931**	0.1390
	15(50)	0.9980	**0.9999**	0.9805	**0.9969**	**0.6719**	0.5931
	15(5)	0.8863	**0.9493**	0.8742	**0.9353**	0.7741	**0.7842**
	15(10)	0.9072	**0.9563**	0.8430	**0.8802**	**0.7435**	0.7184
	15(15)	0.9905	**0.9994**	0.9699	**0.9953**	**0.6466**	0.5406
Δ = 1.50	15(20)	0.9181	**0.9614**	0.8431	**0.8763**	**0.4981**	0.2750
	15(30)	0.9951	**0.9997**	0.9633	**0.9926**	**0.4889**	0.2644
	15(40)	0.9999	**1**	0.9983	**0.9999**	**0.6404**	0.5407
	15(50)	**1**	**1**	0.9998	**1**	0.8730	**0.9122**

Table 6: Type-I-error rate of Friedman test and randomization test under skewed error (lognormal) at α = 0.05

Number of treatments	Number of blocks	$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
		FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	5	**0.0224**	**0.0407**	**0.0426**	**0.0397**	**0.0146**	**0.0399**
	10	**0**	**0.0458**	**0.0031**	**0.0460**	**0.0186**	**0.0464**
	15	**0.0390**	**0.0440**	**0.0096**	**0.0448**	**0**	**0.0454**
$t = 3$	20	**0.0460**	0.0564	**0.0379**	0.0550	**0.0045**	0.0550
	30	**0.0394**	0.0564	**0.0252**	0.0550	**0**	0.0575
	40	**0.0496**	**0.0499**	**0.0449**	0.0505	**0.0249**	**0.0489**
	50	**0.0293**	**0.0480**	**0.0002**	**0.0450**	**0**	**0.0465**
	5	**0.0325**	**0.0271**	**0.0321**	**0.0293**	**0.0160**	**0.0296**
	10	**0.0213**	**0.0372**	**0.0327**	**0.0362**	**0.0085**	**0.0364**
	15	**0.0376**	**0.0397**	**0.0148**	**0.0403**	**0.0034**	**0.0407**
$t = 6$	20	**0.0311**	**0.0373**	**0.0383**	**0.0383**	**0.0314**	**0.0380**
	30	**0.0310**	**0.0310**	**0.0354**	**0.0406**	**0.0136**	**0.0396**
	40	**0.0391**	**0.0382**	**0.0089**	**0.0387**	**0.0051**	**0.0400**
	50	**0.0364**	**0.0409**	**0.0229**	**0.0415**	**0.0060**	**0.0410**
	5	**0.0401**	**0.0406**	**0.0372**	**0.0245**	**0.0420**	**0.0264**
	10	**0.0397**	**0.0395**	**0.0161**	**0.0402**	**00034**	**0.0382**
	15	**0.0356**	**0.0421**	**0.0377**	**0.0413**	**0.0287**	**0.0416**
$t = 9$	20	**0.0351**	**0.0416**	**0.0362**	**0.0408**	**0.0204**	**0.0407**
	30	**0.0464**	**0.0427**	**0.0461**	**0.0447**	**0.0323**	**0.0444**
	40	**0.0445**	**0.0402**	**0.0448**	**0.0407**	**0.0252**	**0.0424**
	50	**0.0331**	**0.0428**	**0.0162**	**0.0435**	**0**	**0.0437**
	5	**0.0259**	**0.0298**	**0.0407**	**0.0307**	**0.0075**	**0.0293**
	10	**0.0427**	**0.0389**	**0.0429**	**0.0392**	**0.0356**	**0.0381**
	15	**0.0388**	**0.0405**	**0.0397**	**0.0410**	**0.0207**	**0.0396**
$t = 12$	20	**00422**	**0.0435**	**0.0085**	**0.0446**	**0.0029**	**0.0428**
	30	**0.0438**	**0.0463**	**0.0456**	**0.0447**	**0.0307**	**0.0446**
	40	**0.0419**	**0.0443**	**0.0467**	**0.0436**	**0.0384**	**0.0430**
	50	**0.0381**	**0.0467**	**0.0439**	**0.0473**	**0.0293**	**0.0474**
	5	**0.0466**	**0.0332**	**0.0441**	**0.0322**	**0.0380**	**0.0324**
	10	**0.0399**	**0.0424**	**0.0411**	**0.0410**	**0.0455**	**0.0405**
	15	**0.0478**	**0.0449**	**0.0475**	**0.0456**	0.0610	**0.0455**
t = 15	20	**0.0422**	**0.0407**	**0.0100**	**0.0403**	**0.0032**	**0.0421**
	30	**0.0406**	**0.0453**	**0.0256**	**0.0451**	**0**	**0.0443**
	40	**0.0449**	**0.0453**	**0.0460**	**0.0444**	**0.0299**	**0.0454**
	50	**0.0434**	**0.0443**	**0.0441**	**0.0437**	**0.0376**	**0.0432**

Table 7: Power of the Friedman test and randomization test under skewed error (lognormal) for t = 3 and 6

α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	3(5)	0.3024	**0.6305**	0.3784	**0.8154**	0.1877	**0.4789**
	3(10)	0.7505	**0.9890**	0.7505	**0.9881**	0.2831	**0.6870**
	3(15)	0.8868	**0.9980**	0.8592	**0.9973**	0.6232	**0.9599**
Δ = 0.50	3(20)	0.5204	**0.9238**	0.5916	**0.9451**	0.2160	**0.5592**
	3(30)	0.8099	**0.9945**	0.8737	**0.9980**	0.2319	**0.6279**
	3(40)	0.6782	**0.9725**	0.4908	**0.9124**	0.3970	**0.8519**
	3(50)	0.7355	**0.9838**	0.6223	**0.9617**	0.2217	**0.5968**
	3(5)	0.3023	**0.6272**	0.3784	**0.8179**	0.1876	**0.4789**
	3(10)	0.8656	**0.9980**	0.8656	**0.9984**	0.5037	**0.9091**
	3(15)	0.8868	**0.9981**	0.8868	**0.9978**	0.6500	**0.9720**
Δ = 0.75	3(20)	0.7755	**0.9891**	0.5916	**0.9475**	0.2351	**0.6188**
	3(30)	0.9616	**0.9998**	0.8840	**0.9976**	0.3970	**0.8391**
	3(40)	0.9589	**0.9996**	0.8656	**0.9967**	0.4909	**0.9114**
	3(50)	0.8907	**0.9981**	0.8002	**0.9916**	0.4900	**0.9065**
	3(5)	0.3784	**0.8110**	0.3024	**0.6311**	0.1507	**0.3098**
	3(10)	0.8655	**0.9977**	0.8655	**0.9982**	0.5990	**0.9562**
	3(15)	0.8937	**0.9986**	0.8868	**0.9976**	0.6938	**0.9777**
Δ = 1.00	3(20)	0.9599	**0.9999**	0.6938	**0.9784**	0.2351	**0.6227**
	3(30)	0.9870	**1**	0.9689	**0.9999**	0.5529	**0.9378**
	3(40)	0.9944	**1**	0.9215	**0.9987**	0.5990	**0.9527**
	3(50)	0.9888	**1**	0.9071	**0.9982**	0.6763	**0.9757**
	3(5)	0.5204	**0.9027**	0.3783	**0.8098**	0.1507	**0.3084**
	3(10)	0.9165	**0.9995**	0.8656	**0.9973**	0.8312	**0.9960**
	3(15)	0.9623	**0.9999**	0.9065	**0.9990**	0.9065	**0.9990**
Δ = 1.50	3(20)	0.9891	**1**	0.9764	**0.9999**	0.3311	**0.7694**
	3(30)	0.9997	**1**	0.9984	**1**	0.9983	**1**
	3(40)	0.9999	**1**	0.9984	**1**	0.8134	**0.9930**
	3(50)	0.9999	**1**	0.9753	**1**	0.9752	**1**
α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	6(5)	0.6364	**0.9149**	0.6364	**0.9142**	0.5904	**0.8832**
	6(10)	0.6284	**0.9056**	0.6316	**0.9061**	0.2242	**0.3103**
	6(15)	0.8625	**0.9943**	0.8625	**0.9935**	0.7427	**0.9710**
Δ = 0.50	6(20)	0.7720	**0.9724**	0.7386	**0.9616**	0.3262	**0.5410**
	6(30)	0.6823	**0.9400**	0.6524	**0.9276**	0.2165	**0.3119**
	6(40)	0.4854	**0.7843**	0.3899	**0.6588**	0.1825	**0.2244**
	6(50)	0.6739	**0.9304**	0.5445	**0.8350**	0.2134	**0.2968**
	6(5)	0.6904	**0.9482**	0.6671	**0.9340**	0.6427	**0.9224**
	6(10)	0.7112	**0.9477**	0.6535	**0.9192**	0.2877	**0.4494**
	6(15)	0.8941	**0.9962**	0.8645	**0.9940**	0.7223	**0.9629**
Δ = 0.75	6(20)	0.8642	**0.9910**	0.7709	**0.9710**	0.4564	**0.7464**
	6(30)	0.7783	**0.9736**	0.7577	**0.9686**	0.2191	**0.3203**
	6(40)	0.7257	**0.9531**	0.4920	**0.7944**	**0.1429**	0.1346
	6(50)	0.8858	**0.9934**	0.7649	**0.9676**	0.3290	**0.5381**
	6(5)	0.6902	**0.9489**	0.6903	**0.9488**	0.6671	**0.9385**
	6(10)	0.8243	**0.9866**	0.7020	**0.9430**	0.2877	**0.4489**
	6(15)	0.9681	**0.9997**	0.8940	**0.9955**	0.7257	**0.9655**
Δ = 1.00	6(20)	0.9480	**0.9988**	0.8845	**0.9930**	0.5430	**0.8412**
	6(30)	0.9677	**0.9997**	0.8149	**0.9799**	0.2837	**0.4661**
	6(40)	0.9505	**0.9987**	0.7051	**0.9422**	**0.1508**	0.1482
	6(50)	0.9761	**0.9996**	0.9223	**0.9971**	0.4926	**0.7757**
	6(5)	0.7232	**0.9629**	0.7232	**0.9636**	0.6903	**0.9504**
	6(10)	0.9336	**0.9988**	0.8715	**0.9945**	0.4329	**0.7001**
	6(15)	0.9947	**1**	0.9713	**0.9996**	0.7292	**0.9652**
Δ = 1.50	6(20)	0.9974	**1**	0.9782	**0.9996**	0.7411	**0.9597**
	6(30)	0.9993	**1**	0.9870	**1**	0.4421	**0.7281**
	6(40)	0.9998	**1**	0.9877	**1**	0.4190	**0.6894**
	6(50)	0.9999	**1**	0.9996	**1**	0.7695	**0.9657**

Table 8: Power of the Friedman test and randomization test under skewed error (lognormal) for t = 9 and 12

α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	9(5)	0.5195	**0.6287**	0.4600	**0.5183**	0.4483	**0.4969**
	9(10)	0.8631	**0.9827**	0.8248	**0.9663**	0.7672	**0.9380**
	9(15)	0.8143	**0.9563**	0.8122	**0.9538**	0.4910	**0.5942**
Δ = 0.50	9(20)	0.6009	**0.7294**	0.5119	**0.6385**	**0.2422**	0.1584
	9(30)	0.8894	**0.9857**	0.8543	**0.9752**	0.3395	**0.3378**
	9(40)	0.5531	**0.7032**	0.4809	**0.5881**	**0.2239**	0.1283
	9(50)	0.6572	**0.8321**	0.6744	**0.8485**	**0.2480**	0.1643
	9(5)	0.5579	**0.6917**	0.5104	**0.6076**	0.4101	**0.4258**
	9(10)	0.8758	**0.9856**	0.8397	**0.9712**	0.7643	**0.9363**
	9(15)	0.8936	**0.9865**	0.8621	**0.9764**	0.5275	**0.6495**
Δ = 0.75	9(20)	0.7022	**0.8751**	0.5679	**0.7191**	**0.2179**	0.1172
	9(30)	0.9758	**0.9995**	0.9694	**0.9996**	0.6043	**0.7755**
	9(40)	0.8102	**0.9517**	0.6036	**0.7694**	**0.2538**	0.1766
	9(50)	0.9188	**0.9906**	0.7848	**0.9373**	0.3694	**0.3865**
	9(5)	0.5835	**0.7289**	0.5074	**0.6020**	0.4356	**0.4732**
	9(10)	0.9138	**0.9945**	0.8705	**0.9823**	0.7691	**0.9370**
	9(15)	0.9440	**0.9966**	0.8986	**0.9871**	0.5675	**0.7063**
Δ = 1.00	9(20)	08470	**0.9699**	0.6617	**0.8335**	**0.2171**	0.1172
	9(30)	0.9974	**1**	0.9920	**1**	0.7434	**0.9108**
	9(40)	0.9626	**0.9979**	0.8539	**0.9729**	**0.2845**	0.2290
	9(50)	0.9943	**1**	0.9435	**0.9955**	0.4518	**0.5318**
	9(5)	0.6191	**0.7832**	0.5722	**0.7116**	0.4672	**0.5281**
	9(10)	0.9744	**0.9995**	0.9297	**0.9959**	0.7928	**0.9509**
	9(15)	0.9789	**0.9993**	0.9692	**0.9986**	0.6367	**0.7972**
Δ = 1.50	9(20)	0.9730	**0.9995**	0.9211	**0.9922**	**0.3495**	0.3462
	9(30)	0.9999	**1**	0.9997	**1**	0.9338	**0.9942**
	9(40)	0.9998	**1**	0.9949	**1**	0.5586	**0.7075**
	9(50)	0.9999	**1**	0.9997	**1**	0.7479	**0.9096**
α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	12(5)	**0.5100**	0.4199	**0.4578**	0.3245	**0.4676**	0.3422
	12(10)	0.8499	**0.9418**	0.8460	**0.9403**	0.7486	**0.8455**
	12(15)	0.7778	**0.8756**	0.7408	**0.8288**	**0.4170**	0.2920
Δ = 0.50	12(20)	**0.4490**	0.3346	**0.5100**	0.4430	**0.3111**	0.1297
	12(30)	0.6149	**0.6380**	0.6273	**0.6594**	**0.2507**	0.0636
	12(40)	0.7963	**0.8846**	0.7916	**0.8800**	**0.4508**	0.3470
	12(50)	0.8284	**0.9239**	0.7100	**0.7895**	**0.3080**	0.1310
	12(5)	**0.5499**	0.4921	**0.5062**	0.4096	**0.4676**	0.3390
	12(10)	0.8735	**0.9609**	0.8621	**0.9517**	0.7607	**0.8596**
	12(15)	0.8288	**0.9246**	0.7823	**0.8803**	**0.4165**	0.2899
Δ = 0.75	12(20)	**0.5800**	0.5658	**0.4459**	0.3270	**0.2885**	0.1015
	12(30)	0.6862	**0.7498**	0.6511	**0.6975**	**0.2256**	0.0423
	12(40)	0.9590	**0.9945**	0.9612	**0.9955**	0.6332	**0.6670**
	12(50)	0.9613	**0.9955**	0.8575	**0.9462**	**0.4058**	0.2803
	12(5)	**0.6147**	0.6133	**0.5575**	0.5066	**0.4854**	0.3726
	12(10)	0.9036	**0.9776**	0.8805	**0.9647**	0.7668	**0.8671**
	12(15)	0.9000	**0.9728**	0.8060	**0.9064**	**0.4712**	0.3830
Δ = 1.00	12(20)	0.7070	**0.7672**	**0.5560**	0.5203	**0.2423**	0.0559
	12(30)	0.8828	**0.9627**	0.7574	**0.8474**	**0.2189**	0.0382
	12(40)	0.9976	**1**	0.9938	**1**	0.8057	**0.8969**
	12(50)	0.9984	**1**	0.9762	**0.9981**	**0.4942**	0.4320
	12(5)	0.7473	**0.8303**	0.6940	**0.7524**	**0.5320**	0.4632
	12(10)	0.9761	**0.9988**	0.9187	**0.9838**	0.7893	**0.8903**
	12(15)	0.9818	**0.9995**	0.9477	**0.9933**	**0.5716**	0.5644
Δ = 1.50	12(20)	0.9582	**0.9957**	0.8444	**0.9288**	**0.2942**	0.1082
	12(30)	0.9964	**0.9999**	0.9494	**0.9939**	**0.2809**	0.0928
	12(40)	0.9999	**1**	0.9999	**1**	0.9612	**0.9946**
	12(50)	0.9999	**1**	0.9999	**1**	0.8341	**0.9281**

Table 9: Power of the Friedman test and randomization test under skewed error (lognormal) for $t = 15$

α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	15(5)	**0.7345**	0.7084	**0.7207**	0.6790	**0.7379**	0.7122
	15(10)	0.8449	**0.8825**	0.8377	**0.8709**	**0.7732**	0.7729
	15(15)	0.7670	**0.7673**	**0.7158**	0.6782	**0.4130**	0.1485
Δ = 0.50	15(20)	**0.6005**	0.4564	**0.6018**	0.4583	**0.3883**	0.1247
	15(30)	**0.6199**	0.5038	**0.5952**	0.4563	**0.3015**	0.0480
	15(40)	**0.6333**	0.5268	**0.5722**	0.4187	**0.3443**	0.0869
	15(50)	0.8593	**0.8944**	0.8530	**0.8874**	**0.4793**	0.2530
	15(5)	**0.7398**	0.7178	**0.7235**	0.6825	**0.7372**	0.7124
	15(10)	0.8384	**0.8726**	0.8242	**0.8536**	**0.7618**	0.7512
	15(15)	0.8592	**0.9035**	0.8492	**0.8902**	**0.4404**	0.1824
Δ = 0.75	15(20)	**0.6620**	0.5719	**0.6454**	0.5401	**0.3816**	0.1148
	15(30)	0.7625	**0.7666**	**0.6852**	0.6231	**0.3207**	0.0617
	15(40)	0.8177	**0.8403**	**0.6869**	0.6266	**0.3619**	0.1024
	15(50)	0.9767	**0.9954**	0.9377	**0.9760**	**0.5902**	0.4407
	15(5)	0.7795	**0.7892**	**0.7438**	0.7224	0.7367	**0.7637**
	15(10)	0.8587	**0.9043**	0.8289	**0.8599**	**0.7432**	0.7201
	15(15)	0.9184	**0.9634**	0.9166	**0.9632**	**0.5085**	0.2911
Δ = 1.00	15(20)	**0.7788**	0.7786	**0.7422**	0.7172	**0.3893**	0.1224
	15(30)	0.8956	**0.9450**	0.8003	**0.8255**	**0.3721**	0.1088
	15(40)	0.9695	**0.9918**	0.8707	**0.9069**	**0.3931**	0.1390
	15(50)	0.9979	**0.9999**	0.9805	**0.9969**	**0.6719**	0.5931
	15(5)	0.8863	**0.9493**	0.8742	**0.9353**	0.7741	**0.7842**
	15(10)	0.9072	**0.9563**	0.8430	**0.8802**	**0.7435**	0.7184
	15(15)	0.9905	**0.9994**	0.9699	**0.9953**	**0.6466**	0.5406
Δ = 1.50	15(20)	0.9181	**0.9614**	0.8431	**0.8763**	**0.4981**	0.2750
	15(30)	0.9951	**0.9997**	0.9633	**0.9926**	**0.4889**	0.2644
	15(40)	0.9999	**1**	0.8431	**0.8763**	**0.6404**	0.5407
	15(50)	0.9999	**1**	0.9997	**1**	0.8730	**0.9122**

Table 10: Type-I-error rate of Friedman test and randomization test under skewed error (Cauchy) at α = 0.05

Number of treatments	Number of blocks	$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
		FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	5	**0.0189**	**0.0374**	**0.0285**	**0.0360**	**0.0187**	**0.0351**
	10	**0.0177**	**0.0466**	**0.0023**	**0.0481**	**0**	**0.0482**
	15	**0**	**0.0465**	**0**	**0.0462**	**0**	**0.0463**
$t = 3$	20	**0.0148**	**0.0489**	**0.0035**	**0.0482**	**0.0002**	0.0513
	30	**0**	0.0542	**0**	0.0583	**0**	0.0567
	40	**0.0458**	0.0514	**0.0458**	0.0512	**0.0443**	0.0502
	50	**0.0344**	**0.0482**	**0.0284**	**0.0491**	**0.0216**	**0.0485**
	5	**0.0047**	**0.0318**	**0.0341**	**0.0308**	**0.0320**	**0.0305**
	10	**0.0221**	**0.0390**	**0.0065**	**0.0381**	**0.0070**	**0.0385**
	15	**0.0017**	**0.0414**	**0.0194**	**0.0389**	**0.0199**	**0.0392**
$t = 6$	20	**0.0027**	**0.0456**	**0.0027**	**0.0347**	**0.0024**	**0.0344**
	30	**0.0133**	**0.0440**	**0.0051**	**0.0442**	**0.0050**	**0.0436**
	40	**0.0346**	**0.0447**	**0.0155**	**0.0411**	**0.0156**	**0.0402**
	50	**0.0251**	**0.0431**	**0.0052**	**0.0402**	**0.0117**	**0.0412**
	5	**0.0047**	**0.0318**	**0.0053**	**0.0317**	**0.0034**	**0.0321**
	10	**0.0221**	**0.0390**	**0.0225**	**0.0409**	**0.0216**	**0.0404**
	15	**0.0017**	**0.0414**	**0.0021**	**0.0422**	**0.0021**	**0.0412**
$t = 9$	20	**0.0027**	**0.0456**	**0.0026**	**0.0467**	**0.0028**	**0.0450**
	30	**0.0133**	**0.0440**	**0.0148**	**0.0437**	**0.0150**	**0.0462**
	40	**0.0346**	**0.0447**	**0.0346**	**0.0461**	**0.0339**	**0.0455**
	50	**0.0251**	**0.0431**	**0.0259**	**0.0435**	**0.0303**	**0.0436**
	5	**0.0033**	**0.0324**	**0.0028**	**0.0323**	**0.0028**	**0.0330**
	10	**0.0084**	**0.0434**	**0.0083**	**0.0434**	**0.0084**	**0.0431**
	15	**0.0033**	**0.0427**	**0.0038**	**0.0416**	**0.0034**	**0.0409**
$t = 12$	20	**0.0164**	**0.0459**	**0.0164**	**0.0451**	**0.0161**	**0.0445**
	30	0.0562	**0.0429**	00562	**0.0416**	0.0564	**0.0427**
	40	**0.0067**	**0.0450**	**0.0012**	**0.0450**	**0.0001**	**0.0456**
	50	**0.0005**	**0.0478**	**0.0006**	**0.0485**	**0.0008**	**0.0478**
	5	**0.0035**	**0.0313**	**0.0034**	**0.0316**	**0.0035**	**0.0313**
	10	**0.0011**	**0.0410**	**0.0012**	**0.0406**	**0.0012**	**0.0413**
	15	**0.0047**	**0.0376**	**0.0052**	**0.0376**	**0.0052**	**0.0376**
t = 15	20	**0.0063**	**0.0451**	**0.0326**	**0.0447**	**0.0329**	**0.0444**
	30	**0.0326**	**0.0425**	**0.0313**	**0..0446**	**0.0332**	**0.0443**
	40	**0.0007**	**0.0447**	**0.0006**	**0.0448**	**0.0006**	**0.0435**
	50	**0.0323**	**0.0463**	**0.0323**	**0.0452**	**0.0311**	**0.0440**

Table 11: Power of the Friedman test and randomization test under skewed error (Cauchy) for t = 3 and 6

α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	3(5)	0.4866	**0.8750**	0.4867	**0.8749**	0.3024	**0.6349**
	3(10)	0.5037	**0.9090**	0.4362	**0.8679**	0.2831	**0.6860**
	3(15)	0.0814	**0.1156**	0.0814	**0.1179**	0.1268	**0.2869**
Δ = 0.50	3(20)	0.1065	**0.2356**	0.1065	**0.2332**	0.0814	**0.1674**
	3(30)	0.1209	**0.3227**	0.2319	**0.6266**	0.1691	**0.4863**
	3(40)	0.2113	**0.5859**	0.2113	**0.5848**	0.0122	**0.2618**
	3(50)	0.2754	**0.7031**	0.2754	**0.7061**	0.1728	**0.4894**
	3(5)	0.4866	**0.8750**	0.4866	**0.8749**	0.3024	**0.6349**
	3(10)	0.4336	**0.8691**	0.5037	**0.9067**	0.2831	**0.6862**
	3(15)	0.0814	**0.1174**	0.0814	**0.1183**	0.0814	**0.1150**
Δ = 0.75	3(20)	0.0814	**0.1645**	0.0733	**0.0926**	0.0814	**0.1685**
	3(30)	0.2831	**0.7205**	0.2831	**0.7154**	0.3088	**0.7508**
	3(40)	0.2113	**0.5813**	0.2113	**0.5850**	0.2208	**0.6118**
	3(50)	0.2217	**0.6009**	0.2319	**0.6410**	0.1728	**0.4901**
	3(5)	0.4866	**0.8735**	0.4866	**0.8750**	0.3024	**0.6349**
	3(10)	0.4336	**0.8664**	0.4336	**0.8673**	0.2831	**0.6862**
	3(15)	**0.0601**	0.0185	0.0814	**0.1159**	0.0601	**0.0152**
Δ = 1.00	3(20)	0.2351	**0.6248**	0.0814	**0.1617**	0.0814	**0.1661**
	3(30)	0.5259	**0.9266**	0.3659	**0.8223**	0.3659	**0.8205**
	3(40)	0.5529	**0.9415**	0.2687	**0.6851**	0.2831	**0.7217**
	3(50)	0.2677	**0.6843**	0.2217	**0.6039**	0.1116	**0.2929**
	3(5)	0.4866	**0.8755**	0.4866	**0.8750**	0.4866	**0.8759**
	3(10)	0.4336	**0.8679**	0.4336	**0.8660**	0.2065	**0.5654**
	3(15)	0.1278	**0.2969**	0.0924	**0.2115**	**0.0601**	0.0174
Δ = 1.50	3(20)	0.3784	**0.8078**	0.3784	**0.8078**	**0.0500**	0
	3(30)	0.8071	**0.9911**	0.8071	**0.9920**	0.4574	**0.8924**
	3(40)	0.6907	**0.9782**	0.4108	**0.8528**	0.3358	**0.7819**
	3(50)	0.3821	**0.8203**	0.2217	**0.6042**	0.1186	**0.3326**
α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	6(5)	0.4349	**0.7054**	0.4349	**0.7042**	0.3211	**0.5071**
	6(10)	0.3607	**0.6011**	0.3201	**0.5222**	0.2717	**0.4231**
	6(15)	**0.1298**	0.0973	**0.1187**	0.0731	**0.1389**	0.1179
Δ = 0.50	6(20)	**0.1412**	0.1284	0.1918	**0.2456**	0.1974	**0.2597**
	6(30)	0.5633	**0.8571**	0.5418	**0.8396**	0.5514	**0.8494**
	6(40)	**0.1025**	0.0493	**0.1033**	0.0514	0.1334	**0.1550**
	6(50)	0.3177	**0.5215**	0.2596	**0.4011**	0.2605	**0.4073**
	6(5)	0.4349	**0.7054**	0.4349	**0.7035**	0.3221	**0.5071**
	6(10)	0.3809	**0.6348**	0.3445	**0.5672**	0.2359	**0.3411**
	6(15)	0.1624	**0.1660**	**0.1187**	0.0730	**0.1435**	0.1289
Δ = 0.75	6(20)	0.1588	**0.1635**	0.1678	**0.1891**	0.2050	**0.2772**
	6(30)	0.6784	**0.9308**	0.6111	**0.8886**	0.6306	**0.9075**
	6(40)	0.1624	**0.1806**	**0.1326**	0.1096	**0.1335**	0.1156
	6(50)	0.5321	**0.8218**	0.4850	**0.7730**	0.2653	**0.4150**
	6(5)	0.4349	**0.7054**	0.4349	**0.7035**	0.3221	**0.5071**
	6(10)	0.3809	**0.6328**	0.3809	**0.6318**	0.2359	**0.3423**
	6(15)	0.1918	**0.2299**	0.1576	**0.1563**	**0.1435**	0.1261
Δ = 1.00	6(20)	0.1955	**0.2466**	0.1732	**0.1986**	0.2012	**0.2705**
	6(30)	0.7593	**0.9620**	0.6616	**0.9197**	0.6463	**0.9162**
	6(40)	0.1974	**0.2593**	0.2330	**0.3462**	**0.1421**	0.1349
	6(50)	0.7626	**0.9637**	0.6952	**0.9374**	0.3525	**0.5821**
	6(5)	0.4349	**0.7058**	0.3708	**0.5961**	0.3221	**0.5064**
	6(10)	0.3647	**0.6030**	0.3809	**0.6313**	0.2557	**0.3856**
	6(15)	0.5847	**0.8686**	0.4150	**0.6752**	0.1818	**0.2135**
Δ = 1.50	6(20)	0.3728	**0.6046**	0.2031	**0.2639**	0.2088	**0.2869**
	6(30)	0.9190	**0.9970**	0.8249	**0.9821**	0.6889	**0.9373**
	6(40)	0.5202	**0.7948**	0.4378	**0.7065**	0.2022	**0.2759**
	6(50)	0.9816	**1**	0.8967	**0.9945**	0.5763	**0.8610**

Table 12: Power of the Friedman test and randomization test under skewed error (Cauchy) for t = 9 and 12

α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	9(5)	0.4101	**0.4476**	0.4101	**0.4493**	**0.3552**	0.3412
	9(10)	0.4133	**0.4569**	0.4277	**0.4849**	**0.3552**	0.3470
	9(15)	0.3897	**0.4157**	0.5354	**0.6665**	0.5165	**0.6328**
Δ = 0.50	9(20)	0.4944	**0.6064**	0.4602	**0.5481**	0.4882	**0.5944**
	9(30)	0.5896	**0.7519**	0.5472	**0.6982**	0.6174	**0.7868**
	9(40)	0.6681	**0.8390**	0.6346	**0.8063**	0.2207	**0.6118**
	9(50)	**0.3056**	0.2691	**0.2998**	0.2575	0.5981	**0.7629**
	9(5)	0.3875	**0.4046**	0.3875	**0.4044**	**0.3552**	0.3412
	9(10)	0.4293	**0.4831**	0.4531	**0.5306**	**0.3552**	0.3470
	9(15)	**0.3315**	0.3042	0.4314	**0.4968**	0.5164	**0.6328**
Δ = 0.75	9(20)	0.5608	**0.7013**	0.4943	**0.6020**	0.4882	**0.5944**
	9(30)	0.6879	**0.8612**	0.6960	**0.8695**	0.6174	**0.7868**
	9(40)	0.7835	**0.9350**	0.7335	**0.8999**	0.6841	**0.8552**
	9(50)	**0.3425**	0.3382	**0.3294**	0.3123	**0.2826**	0.2261
	9(5)	**0.3617**	0.3527	0.3875	**0.4054**	**0.3552**	0.3412
	9(10)	0.5209	**0.6460**	0.4531	**0.5291**	0.3827	**0.3980**
	9(15)	**0.2500**	0.1599	0.4133	**0.4630**	0.4704	**0.5600**
Δ = 1.00	9(20)	0.5968	**0.7477**	0.5395	**0.6696**	0.5217	**0.6477**
	9(30)	0.7899	**0.9380**	0.7515	**0.9132**	0.6847	**0.8614**
	9(40)	0.8963	**0.9857**	0.8757	**0.9800**	0.7252	**0.8931**
	9(50)	0.4591	**0.5432**	0.3875	**0.4201**	**0.2712**	0.2060
	9(5)	**0.3552**	0.3421	**0.3552**	0.3419	**0.3358**	0.3020
	9(10)	0.5447	**0.6814**	0.5164	**0.6358**	0.3827	**0.3965**
	9(15)	**0.1966**	0.0788	**0.2114**	0.1018	0.4026	**0.4414**
Δ = 1.50	9(20)	0.7459	**0.9083**	0.6554	**0.8179**	0.5359	**0.6682**
	9(30)	0.9727	**0.9993**	0.9317	**0.9945**	0.8122	**0.9547**
	9(40)	0.9887	**0.9997**	0.9687	**0.9978**	0.8534	**0.9716**
	9(50)	0.8019	**0.9450**	0.5841	**0.7363**	**0.3290**	0.3091
α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	12(5)	0.6451	**0.6795**	0.6531	**0.6973**	0.6410	**0.6759**
	12(10)	**0.4265**	0.2954	**0.4331**	0.3079	**0.3848**	0.2268
	12(15)	**0.5105**	0.4456	**0.4719**	0.3774	**0.4967**	0.4218
Δ = 0.50	12(20)	**0.5353**	0.4906	**0.4919**	0.4168	**0.4846**	0.4039
	12(30)	0.7331	**0.8131**	0.7009	**0.7673**	0.7276	**0.8055**
	12(40)	0.7617	**0.8477**	0.7973	**0.8909**	0.7734	**0.8653**
	12(50)	0.8079	**0.9026**	0.7679	**0.8583**	0.7152	**0.7936**
	12(5)	**0.5976**	0.5974	0.6218	**0.6428**	0.6231	**0.6416**
	12(10)	**0.4231**	0.2894	**0.4182**	0.2850	**0.3764**	0.2134
	12(15)	**0.5340**	0.4881	**0.4945**	0.4164	**0.5046**	0.4356
Δ = 0.75	12(20)	**0.5730**	0.5541	**0.5380**	0.4931	**0.5098**	0.4453
	12(30)	0.8366	**0.9254**	0.8107	**0.9011**	0.7569	**0.8427**
	12(40)	0.9360	**0.9894**	0.9128	**0.9795**	0.8207	**0.9111**
	12(50)	0.8620	**0.9491**	0.8453	**0.9350**	0.7634	**0.8543**
	12(5)	**0.5859**	0.5783	0.6218	**0.6425**	0.6231	**0.6440**
	12(10)	**0.3731**	0.2089	**0.3714**	0.2068	**0.3606**	0.1903
	12(15)	**0.5888**	0.5887	**0.5335**	0.4889	**0.5131**	0.4524
Δ = 1.00	12(20)	0.6105	**0.6192**	**0.5534**	0.5205	**0.5357**	0.4874
	12(30)	0.9064	**0.9757**	0.9008	**0.9741**	0.8037	**0.8949**
	12(40)	0.9836	**0.9996**	0.9629	**0.9972**	0.8952	**0.9694**
	12(50)	0.9621	**0.9965**	0.9507	**0.9940**	0.8108	**0.9037**
	12(5)	**0.5015**	0.4201	**0.5771**	0.5618	**0.5874**	0.5787
	12(10)	**0.4098**	0.2665	**0.3748**	0.2130	**0.3497**	0.1743
	12(15)	0.6769	**0.7335**	0.6259	**0.6534**	**0.5474**	0.5153
Δ = 1.50	12(20)	0.7268	**0.8037**	0.6869	**0.7427**	**0.5640**	0.5374
	12(30)	0.9797	**0.9985**	0.9768	**0.9982**	0.9047	**0.9748**
	12(40)	**0.7706**	0.7696	0.9985	**1**	0.9501	**0.9939**
	12(50)	**0.7318**	0.7059	0.9917	**0.9998**	0.9108	**0.9786**

Table 13: Power of the Friedman test and randomization test under skewed error (Cauchy) for $t = 15$

α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	15(5)	**0.6728**	0.5822	**0.6450**	0.5223	**0.6226**	0.4795
	15(10)	**0.7539**	0.7414	**0.6911**	0.6242	**0.6812**	0.6070
	15(15)	**0.4510**	0.2031	**0.4104**	0.1473	**0.4411**	0.1856
Δ = 0.50	15(20)	**0.7703**	0.7677	**0.6057**	0.4666	**0.6117**	0.4778
	15(30)	**0.6072**	0.4626	**0.5932**	0.4407	**0.6581**	0.5599
	15(40)	**0.6216**	0.5014	**0.6405**	0.5382	**0.6250**	0.5078
	15(50)	**0.6930**	0.6385	**0.6740**	0.6030	0.3283	**0.6345**
	15(5)	**0.5015**	0.4201	**0.6498**	0.5323	**0.6226**	0.4799
	15(10)	**0.4098**	0.2665	**0.7162**	0.6742	**0.6766**	0.5964
	15(15)	0.6770	**0.7335**	0.4259	0.1682	0.4302	0.1702
Δ = 0.75	15(20)	0.7268	**0.8037**	**0.6768**	0.6035	**0.6238**	0.5015
	15(30)	**0.6478**	0.5379	**0.5933**	0.4411	**0.6358**	0.5192
	15(40)	**0.7706**	0.7696	**0.7320**	0.7046	**0.6595**	0.5739
	15(50)	**0.7318**	0.7059	**0.6872**	0.6294	**0.6831**	0.6226
	15(5)	**0.6728**	0.5822	**0.6482**	0.5280	**0.6276**	0.4901
	15(10)	**0.7539**	0.7414	**0.7214**	0.6848	**0.7308**	0.7031
	15(15)	**0.4510**	0.2031	**0.4418**	0.1883	**0.4533**	0.2030
Δ = 1.00	15(20)	**0.7703**	0.7677	**0.7399**	0.7159	**0.6490**	0.5524
	15(30)	**0.6856**	0.6090	**0.6206**	0.4885	**0.6605**	0.5651
	15(40)	**0.6216**	0.5014	0.8295	**0.8595**	**0.6980**	0.6429
	15(50)	**0.6930**	0.6385	**0.7448**	0.7301	**0.7152**	0.6779
	15(5)	**0.6873**	0.6102	**0.6658**	0.5664	**0.7076**	0.6554
	15(10)	0.7865	**0.7982**	**0.7358**	0.7085	**0.7076**	0.6554
	15(15)	**0.5468**	0.3531	**0.4958**	0.2710	**0.4457**	0.1934
Δ = 1.50	15(20)	0.8638	**0.9068**	0.8163	**0.8384**	**0.7400**	0.7164
	15(30)	0.8621	**0.9037**	**0.7607**	0.7495	**0.6440**	0.5349
	15(40)	0.9845	**0.9983**	0.9504	**0.9842**	0.7957	**0.8090**
	15(50)	0.9585	**0.9893**	0.8668	**0.9092**	**0.7169**	0.6835

Table 14: Type-I-error rate of Friedman test and randomization test under skewed error (logistic) at α = 0.05

Number of	Number	$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
treatments	of blocks	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	5	0.0519	**0.0363**	**0.0452**	**0.0382**	**0.0419**	**0.0344**
	10	**0.0465**	**0.0443**	**0.0458**	**0.0450**	**0.0441**	**0.0458**
	15	**0.0487**	**0.0454**	0.0505	**0.0477**	**0.0480**	**0.0475**
$t = 3$	20	0.0504	0.0506	0.0504	0.0510	0.0503	0.0524
	30	**0.0493**	0.0538	**0.0491**	0.0554	0.0501	0.0509
	40	**0.0492**	**0.0442**	**0.0498**	**0.0463**	0.0511	**0.0426**
	50	0.0507	**0.0496**	**0.0494**	**0.0480**	0.0500	0.0506
	5	**0.0424**	**0.0297**	**0.0435**	**0.0298**	**0.0428**	**0.0324**
	10	**0.0407**	**0.0367**	**0.0436**	**0.0365**	**0.0432**	**0.0347**
	15	**0.0450**	**0.0371**	**0.0494**	**0.0392**	**0.0468**	**0.0374**
$t = 6$	20	**0.0402**	**0.0383**	**0.0430**	**0.0388**	**0.0447**	**0.0394**
	30	**0.0414**	**0.0421**	**0.0430**	**0.0424**	**0.0430**	**0.0422**
	40	**0.0379**	**0.0414**	**0.0382**	**0.0409**	**0.0401**	**00406**
	50	**0.0467**	**0.0386**	0.0501	**0.0376**	**0.0492**	**0.0371**
	5	**0.0411**	**0.0298**	**0.0355**	**0.0294**	**0.0357**	**0.0283**
	10	**0.0442**	**0.0373**	**0.0445**	**00373**	**0.0436**	**0.0375**
	15	**0.0467**	**0.0418**	**0.0489**	**0.0414**	**0.0481**	**0.0416**
$t = 9$	20	**00433**	**0.0389**	**0.0453**	**0.0409**	**0.0460**	**0.0401**
	30	**0.0455**	**0.0440**	**0.0450**	**0.0437**	**0.0462**	**0.0462**
	40	**0.0471**	**0.0438**	**0.0473**	**0.0430**	**0.0492**	**0.0435**
	50	**0.0437**	**0.0438**	**0.0439**	**0.0427**	**0.0452**	**0.0443**
	5	**0.0437**	**0.0331**	**0.0444**	**0.0322**	**0.0443**	**0.0302**
	10	**0.0481**	**0.0422**	**0.0485**	**0.0412**	**0.0492**	**0.0415**
	15	**0.0453**	**0.0421**	**0.0440**	**0.0434**	**0.0458**	**0.0439**
$t = 12$	20	**0.0492**	**0.0470**	**0.0492**	**0.0469**	**0.0488**	**0.0480**
	30	**0.0446**	**0.0481**	**0.0459**	**0.0477**	**0.0443**	**0.0474**
	40	**0.0491**	**0.0403**	**0.0491**	**0.0401**	**0.0482**	**0.0394**
	50	**0.0494**	**0.0403**	**0.0494**	**0.0399**	0.0507	**0.0420**
	5	0.0524	**0.0309**	0.0520	**0.0301**	0.0520	**0.0303**
	10	**0.0475**	**0.0415**	**0.0447**	**0.0404**	**0.0454**	**0.0402**
	15	**0.0453**	**0.0382**	**0.0454**	**0.0395**	**0.0484**	**0.0397**
t = 15	20	**0.0461**	**0.0412**	**0.0451**	**0.0399**	**0.0451**	**0.0391**
	30	**0.0460**	**0.0450**	**0.0450**	**0.0447**	**0.0450**	**0.0455**
	40	**0.0471**	**0.0435**	**0.0490**	**0.0449**	**0.0485**	**0.0450**
	50	**0.0469**	**0.0445**	**0.0449**	**0.0439**	**0.0454**	**0.0449**

Table 15: Power of the Friedman test and randomization test under skewed error (logistic) for t = 3 and 6

α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	3(5)	0.5204	**0.9110**	0.5204	**0.9095**	0.4866	**0.8774**
	3(10)	0.5529	**0.9362**	0.5529	**0.9324**	0.2065	**0.5693**
	3(15)	0.6233	**0.9556**	0.8208	**0.9941**	0.7252	**0.9824**
Δ = 0.50	3(20)	0.9058	**0.9987**	0.7849	**0.9918**	0.4952	**0.9076**
	3(30)	0.8656	**0.9966**	0.6729	**0.9735**	0.4396	**0.8810**
	3(40)	0.3263	**0.7592**	0.2208	**0.6141**	0.0980	**0.2225**
	3(50)	0.8705	**0.9972**	0.9229	**0.9987**	0.8105	**0.9934**
	3(5)	0.3024	**0.6272**	0.3784	**0.8179**	0.4866	**0.8785**
	3(10)	0.8656	**0.9980**	0.8656	**0.9984**	0.2065	**0.5675**
	3(15)	0.8868	**0.9981**	0.8868	**0.9978**	0.7252	**0.9820**
Δ = 0.75	3(20)	0.7755	**0.9891**	0.5916	**0.9475**	0.4426	**0.8821**
	3(30)	0.9616	**0.9998**	0.8850	**0.9976**	0.4396	**0.8814**
	3(40)	0.9589	**0.9996**	0.8656	**0.9967**	0.1283	**0.3395**
	3(50)	0.9589	**0.9996**	0.8907	**0.9981**	0.8429	**0.9960**
	3(5)	0.5204	**0.9110**	0.5204	**0.9119**	0.4866	**0.8785**
	3(10)	0.5368	**0.9260**	0.5529	**0.9346**	0.2065	**0.5675**
	3(15)	0.8937	**0.9988**	0.8207	**0.9940**	0.8208	**0.9941**
Δ = 1.00	3(20)	0.9058	**0.9986**	0.8492	**0.9958**	0.5204	**0.9290**
	3(30)	0.9256	**0.9990**	0.8656	**0.9972**	0.4923	**0.9052**
	3(40)	0.4736	**0.8991**	0.6521	**0.9695**	0.1553	**0.4428**
	3(50)	0.9505	**0.9992**	0.9473	**0.9996**	0.8429	**0.9959**
	3(5)	0.7399	**0.9915**	0.5200	**0.9108**	0.5204	**0.9091**
	3(10)	0.6686	**0.9714**	0.5529	**0.9348**	0.2831	**0.6898**
	3(15)	0.9841	**1**	0.8937	**0.9990**	0.8937	**0.9985**
Δ = 1.50	3(20)	0.9599	**0.9999**	0.9599	**0.9998**	0.6877	**0.9797**
	3(30)	0.9468	**0.9996**	0.9359	**0.9889**	0.6729	**0.9714**
	3(40)	0.9605	**0.9997**	0.8071	**0.9936**	0.2065	**0.5572**
	3(50)	0.9954	**1**	0.9869	**1**	0.8843	**0.9974**
α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	6(5)	0.4110	**0.6695**	0.4110	**0.6724**	0.3789	**0.6202**
	6(10)	0.8206	**0.9884**	0.8168	**0.9883**	0.7398	**0.9652**
	6(15)	0.6247	**0.9083**	0.5520	**0.8554**	0.5915	**0.8878**
Δ = 0.50	6(20)	0.8280	**0.9865**	0.8224	**0.9853**	0.5644	**0.8590**
	6(30)	0.8442	**0.9878**	0.8286	**0.9869**	0.7976	**0.9806**
	6(40)	0.5257	**0.8304**	0.5294	**0.8315**	0.4230	**0.7027**
	6(50)	0.5880	**0.8855**	0.6002	**0.8926**	0.5666	**0.8709**
	6(5)	0.4110	**0.6670**	0.4110	**0.6715**	0.3789	**0.6210**
	6(10)	0.8369	**0.9914**	0.8168	**0.9881**	0.7497	**0.9690**
	6(15)	0.7477	**0.9656**	0.5662	**0.8639**	0.5544	**0.8563**
Δ = 0.75	6(20)	0.8627	**0.9920**	0.8437	**0.9889**	0.5644	**0.8581**
	6(30)	0.8519	**0.9901**	0.8298	**0.9869**	0.7716	**0.9739**
	6(40)	0.6512	**0.9261**	0.5618	**0.8632**	0.4010	**0.6730**
	6(50)	0.7312	**0.9597**	0.5975	**0.8898**	0.5517	**0.8576**
	6(5)	0.3950	**0.6404**	0.4110	**0.6681**	0.3789	**0.6201**
	6(10)	0.8627	**0.9937**	0.8386	**0.9905**	0.7497	**0.9690**
	6(15)	0.7525	**0.9672**	0.5961	**0.8838**	0.5847	**0.8801**
Δ = 1.00	6(20)	0.8771	**0.9942**	0.8589	**0.9916**	0.5921	**0.8827**
	6(30)	0.8885	**0.9942**	0.8369	**0.9870**	0.7739	**0.9745**
	6(40)	0.6974	**0.9460**	0.6228	**0.9079**	0.4070	**0.6816**
	6(50)	0.8202	**0.9841**	0.7101	**0.9499**	0.5409	**0.8501**
	6(5)	0.5556	**0.8516**	0.4506	**0.7227**	0.3869	**0.6292**
	6(10)	0.8864	**0.9971**	0.8926	**0.9973**	0.7969	**0.9815**
	6(15)	0.8120	**0.9804**	0.7007	**0.9461**	0.6289	**0.9095**
Δ = 1.50	6(20)	0.9558	**0.9992**	0.9260	**0.9982**	0.6656	**0.9298**
	6(30)	0.9818	**0.9998**	0.9405	**0.9983**	0.8200	**0.9844**
	6(40)	0.9199	**0.9973**	0.8051	**0.9802**	0.4269	**0.7104**
	6(50)	0.9924	**1**	0.8365	**0.9850**	0.6322	**0.9160**

Table 16: Power of the Friedman test and randomization test under skewed error (logistic) for t = 9 and 12

α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	9(5)	0.7485	**0.9283**	0.7255	**0.9084**	0.7866	**0.9567**
	9(10)	0.7382	**0.9125**	0.7033	**0.8832**	0.7090	**0.8856**
	9(15)	0.8028	**0.9522**	0.8667	**0.9804**	0.8122	**0.9583**
Δ = 0.50	9(20)	0.9116	**0.9914**	0.9167	**0.9927**	0.8528	**0.9742**
	9(30)	0.8721	**0.9810**	0.8382	**0.9696**	0.7131	**0.8881**
	9(40)	0.4297	**0.4878**	0.4137	**0.4584**	**0.3532**	0.3456
	9(50)	0.7543	**0.9226**	0.7989	**0.9517**	0.7148	**0.8918**
	9(5)	0.7485	**0.9273**	0.7255	**0.9089**	0.7866	**0.9575**
	9(10)	0.7485	**0.9207**	0.7329	**0.9078**	0.7010	**0.8800**
	9(15)	0.8348	**0.9683**	0.8687	**0.9807**	0.8373	**0.9690**
Δ = 0.75	9(20)	0.9386	**0.9963**	0.9240	**0.9943**	0.8750	**0.9822**
	9(30)	0.8752	**0.9830**	0.8615	**0.9770**	0.7308	**0.9031**
	9(40)	0.4535	**0.5316**	0.4133	**0.4615**	**0.3072**	0.2637
	9(50)	0.9134	**0.9913**	0.8709	**0.9802**	0.7745	**0.9353**
	9(5)	0.7549	**0.9316**	0.7298	**0.9120**	0.7866	**0.9565**
	9(10)	0.7575	**0.9238**	0.7555	**0.9248**	0.6930	**0.8719**
	9(15)	0.8617	**0.9778**	0.8936	**0.9869**	0.8392	**0.9695**
Δ = 1.00	9(20)	0.9476	**0.9971**	0.9196	**0.9933**	0.8726	**0.9817**
	9(30)	0.9204	**0.9945**	0.8819	**0.9847**	0.7498	**0.9160**
	9(40)	0.4948	**0.6009**	0.4428	**0.5107**	**0.2992**	0.2503
	9(50)	0.9699	**0.9987**	0.9236	**0.9928**	0.8172	**0.9583**
	9(5)	0.8548	**0.9843**	0.8039	**0.9614**	0.8073	**0.9688**
	9(10)	0.8474	**0.9746**	0.7784	**0.9379**	0.7112	**0.8882**
	9(15)	0.9545	**0.9983**	0.9282	**0.9947**	0.8526	**0.9745**
Δ = 1.50	9(20)	0.9810	**0.9997**	0.9703	**0.9987**	0.8914	**0.9871**
	9(30)	0.9765	**0.9988**	0.9480	**0.9968**	0.7971	**0.9457**
	9(40)	0.7902	**0.9379**	0.5311	**0.6585**	**0.2944**	0.2401
	9(50)	0.9962	**0.9998**	0.9871	**0.9994**	0.8868	**0.9845**
α = 0.05		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	12(5)	**0.8684**	**0.9663**	**0.9008**	**0.9829**	**0.9087**	**0.9860**
	12(10)	0.7437	**0.8272**	0.7383	**0.8202**	0.7581	**0.8454**
	12(15)	**0.9090**	**0.9798**	**0.9142**	**0.9809**	**0.8871**	**0.9668**
Δ = 0.50	12(20)	**0.9066**	**0.9766**	**0.9087**	**0.9776**	**0.8410**	**0.9335**
	12(30)	**0.8177**	**0.9134**	**0.8015**	**0.8977**	0.6276	0.6634
	12(40)	0.5058	0.4482	0.5140	0.4650	0.4882	0.4113
	12(50)	0.6788	0.7450	0.6693	0.7324	0.6164	0.6513
	12(5)	0.9008	**0.9832**	0.9058	**0.9851**	0.9254	**0.9923**
	12(10)	0.7754	**0.8656**	0.7426	**0.8264**	0.7591	**0.8474**
	12(15)	0.9154	**0.9819**	0.9174	**0.9828**	0.8968	**0.9739**
Δ = 0.75	12(20)	0.9128	**0.9806**	0.8987	**0.9740**	0.8082	**0.9034**
	12(30)	0.8394	**0.9317**	0.8010	**0.8992**	0.6276	**0.6634**
	12(40)	**0.5179**	0.4695	**0.4688**	0.3855	**0.4635**	0.3678
	12(50)	0.7821	**0.8810**	0.7092	**0.7890**	0.6432	**0.6937**
	12(5)	0.9058	**0.9850**	0.9082	**0.9859**	0.9334	**0.9947**
	12(10)	0.7817	**0.8740**	0.7470	**0.8334**	0.7591	**0.8467**
	12(15)	0.9412	**0.9912**	0.9288	**0.9872**	0.9008	**0.9754**
Δ = 1.00	12(20)	0.9188	**0.9830**	0.9154	**0.9825**	0.8136	**0.9074**
	12(30)	0.8683	**0.9538**	0.8146	**0.9090**	0.5983	**0.6148**
	12(40)	0.6146	**0.6407**	**0.4788**	0.4035	**0.4142**	0.2873
	12(50)	0.8590	**0.9465**	0.7589	**0.8543**	0.6514	**0.7062**
	12(5)	0.9370	**0.9952**	0.9423	**0.9961**	0.9383	**0.9958**
	12(10)	0.8418	**0.9342**	0.7841	**0.8777**	0.7758	**0.8661**
	12(15)	0.9900	**0.9997**	0.9650	**0.9975**	0.9081	**0.9797**
Δ = 1.50	12(20)	0.9629	**0.9960**	0.9414	**0.9913**	0.8218	**0.9145**
	12(30)	0.9656	**0.9967**	0.8682	**0.9530**	0.6374	**0.6797**
	12(40)	0.8052	**0.8937**	0.6548	**0.7007**	**0.3954**	0.2598
	12(50)	0.9898	**0.9997**	0.9022	**0.9740**	0.7127	**0.7959**

Table 17: Power of the Friedman test and randomization test under skewed error (logistic) for $t = 15$

$\alpha = 0.05$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:1:1.5$		$\sigma_1^2 : \sigma_2^2 : \sigma_3^2 = 1:2:3$	
Effect size	t(b)	FR-Test	R-Test	FR-Test	R-Test	FR-Test	R-Test
	15(5)	0.8806	**0.9407**	0.8891	**0.9502**	0.9070	**0.9656**
	15(10)	0.9160	**0.9639**	0.9058	**0.9557**	0.9225	**0.9710**
	15(15)	0.8965	**0.9431**	0.8835	**0.9284**	0.8608	**0.9044**
$\Delta = 0.50$	15(20)	0.8860	**0.9343**	0.8791	**0.9265**	**0.7555**	0.7454
	15(30)	0.8560	**0.8956**	0.8161	**0.8401**	**0.6541**	0.5612
	15(40)	**0.6467**	0.5439	**0.6264**	0.5077	**0.6157**	0.4909
	15(50)	0.7782	**0.7861**	**0.7644**	0.7627	**0.7310**	0.7068
	15(5)	0.8964	**0.9563**	0.8964	**0.9573**	0.9111	**0.9699**
	15(10)	0.9274	**0.9747**	0.9247	**0.9734**	0.9177	**0.9674**
	15(15)	0.8979	**0.9465**	0.8770	**0.9218**	0.8368	**0.8752**
$\Delta = 0.75$	15(20)	0.8951	**0.9425**	0.8783	**0.9249**	**0.7555**	0.7461
	15(30)	0.9211	**0.9648**	0.8791	**0.9257**	**0.6745**	0.5965
	15(40)	**0.6303**	0.5118	**0.6157**	0.4901	**0.5741**	0.4154
	15(50)	0.8497	**0.8885**	0.8388	**0.8740**	**0.7639**	0.7591
	15(5)	0.9119	**0.9717**	0.9049	**0.9653**	0.9099	**0.9685**
	15(10)	0.9374	**0.9813**	0.9254	**0.9723**	0.9246	**0.9724**
	15(15)	0.9133	**0.9607**	0.8608	**0.9044**	0.8304	**0.8674**
$\Delta = 1.00$	15(20)	0.9138	**0.9138**	0.8858	**0.9331**	**0.7563**	0.7469
	15(30)	0.9519	**0.9849**	0.9431	**0.9805**	**0.7225**	0.6871
	15(40)	**0.7006**	0.6398	**0.6259**	0.5064	**0.5633**	0.3945
	15(50)	0.9490	**0.9834**	0.9403	**0.9796**	0.8113	**0.8357**
	15(5)	0.9276	**0.9839**	0.9128	**0.9741**	0.9214	**0.9793**
	15(10)	0.9665	**0.9938**	0.9541	**0.9899**	0.9345	**0.9791**
	15(15)	0.9433	**0.9814**	0.9136	**0.9610**	0.8328	**0.8699**
$\Delta = 1.50$	15(20)	0.9503	**0.9855**	0.9068	**0.9545**	**0.7603**	0.7532
	15(30)	0.9886	**0.9992**	0.9862	**0.9990**	0.8455	**0.8839**
	15(40)	0.8899	**0.9330**	**0.7734**	0.7721	**0.5589**	0.3884
	15(50)	0.9963	**0.9999**	0.9833	**0.9985**	0.8966	**0.9426**